Plug-and-play voltage and frequency control of islanded microgrids with meshed topology

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Abstract—In this paper we propose a new decentralized control scheme for Islanded microGrids (ImGs) composed by the interconnection of Distributed Generation Units (DGUs). Local controllers regulate voltage and frequency at the Point of Common Coupling (PCC) of each DGU and they are able to guarantee stability of the overall ImG. The control design procedure is decentralized, since, besides two global scalar quantities, the synthesis of a local controller uses only information on the corresponding DGU and lines connected to it. Most important, our design procedure enables Plug-and-Play (PnP) operations: when a DGU is plugged in or out, only DGUs physically connected to it have to retune their local controllers. We study the performance of the proposed controllers simulating different scenarios in Matlab/Simulink and using performance indexes proposed in IEEE standards.

I. INTRODUCTION

In recent years, research on Islanded microGrids (ImG) has received major attention. ImGs are self-sufficient micro grids composed of several Distributed Generation Units (DGUs) designed to operate safely and reliably in absence of a connection with the main grid. Besides fostering the use of renewable generation, ImGs bring distributed generation sources close to loads and allow power to be delivered to rural areas, remote lands, islands or harsh environments [1], [2], [3]. The interest in ImGs is also motivated by microgrids that normally operate in grid-connected mode and that can be switched off-grid for guaranteeing users remain powered in presence of grid faults. In particular, for buildings such hospitals and airports, ImGs offer a interesting solution for emergency generation since, differently from common diesel generators, power is produced and delivered to the main grid in absence of faults.

For grid-connected microgrids, voltage and frequency are set by the main grid. However, in islanded mode, voltage and frequency control must be operated by DGUs. This is a challenging task, especially if one allows for (a) meshed topology with the goal increasing redundancy and robustness to line faults; (b) decentralized regulation of voltage and frequency, meaning that each DGU is equipped with a local controller and controllers do not communicate in real-time.

As reviewed in [3] many available decentralized regulators are based on droop control [4], [5], [6], [7], [8], [9]. The main drawback of applying the droop method to ImGs is that frequency and amplitude deviations can be heavily affected by loads. For these reasons, a secondary control layer to restore system frequency and voltage to nominal values is needed [4], [10], [11]. Stability is another critical issue in ImGs controlled in a decentralized way [3]. The key challenge is to guarantee stability is not spoiled by the interaction among DGUs and, in the context of droop control, this issue has been investigated only recently [12]. For regulators not based on droop control, almost all studies focused on radial ImGs (i.e. DGUs are not connected in a loop fashion) while control of ImGs with meshed topology is still largely unexplored [3].

In this paper we consider the design of decentralized regulators for meshed ImGs with a view on decentralization of the synthesis procedure. More specifically, we develop a Plug-and-Play (PnP) design algorithm where the synthesis of a local controller for a DGU requires parameters of transmission lines connected to it, the knowledge of two global scalar parameters, but not specific information about any other DGU. This implies that when a DGU is plugged in or out, only DGUs physically connected to it have to retune their local controllers.

PnP control design for general linear constrained systems has been proposed in [13], [14] and [15]. PnP design for ImGs is however different since it is based on the concept of neutral interactions [16] rather than on robustness against subsystem coupling. Furthermore, for achieving neutral interactions among DGUs, we exploit Quasi-Stationary Line (QSL) approximations of line dynamics [17].

Our theoretical results are backed up by simulations using realistic models of Voltage Source Converters (VSCs), associated filters and transformers. As a first testbed, we consider two radially connected DGUs [18] and show that, in spite of QSL approximations, PnP controllers exhibit very good performances in terms of voltage tracking and robustness to nonlinear and unbalanced loads (in the last two cases, indices from the IEEE standards [19] have been used). We then consider a meshed ImG with 10 DGUs including loops and discuss the real-time plugging in and out of a DGU.

The paper is organized as follows. In Section II we present dynamical models of ImGs and introduce the adopted line approximation. In Section III we exploit the notion of neutral interactions for designing decentralized controllers and we discuss how to perform PnP operations. In Section IV we study performance of PnP controllers through simulation case studies. Section V is devoted to some conclusions.

II. MICROGRID MODEL

In this section, we present dynamical models of ImGs used in this paper. For the sake of clearness, we first introduce an ImG consisting of two parallel DGUs and then generalize
for the line current and introducing a single state variable. For a definition of expansion of a system we defer the reader to Section 3.4 in [16].

In the next section, we propose an approximate model that allows one to describe each DGU as a dynamical system affected directly by state of the other DGU, hence avoiding the need of using the line current in the DGU state equations.

A. QSL model

As in equation (T1.10) in [17], we set \( \frac{dI_{i,i,dq}}{dt} = 0 \) and \( \frac{dI_{i,i,dq}}{dt} = 0 \) (see also [24] and references therein). Then, (2) gives the QSL model

\[
\dot{I}_{s,o,dq} = \frac{V_{o,dq}}{(R_{s,o} + j\omega_0 L_{s,o})} - \frac{V_{s,dq}}{(R_{s,o} + j\omega_0 L_{s,o})}
\]

We then replace variables \( I_{s,o,dq} \) in (1a) with the right-hand side of (3). Splitting complex \( dq \) quantities in their \( d \) and \( q \) components one obtains the following model of DGU \( * \) (namely \( \Sigma^{DGU}_{d,q} \))

\[
\dot{x}_{[s]}(t) = A_{s,s}[x_{[s]}(t)] + B_{s,u_{[s]}(t)} + M_{s,d_{[s]}(t)} + \xi_{[s]}(t)
\]

\[
y_{[s]}(t) = C_{s,x_{[s]}(t)}
\]

\[
z_{[s]}(t) = H_{s,y_{[s]}(t)}
\]

For the line \( s \) we obtain

\[
\frac{dI_{s,o,dq}}{dt} + j\omega_0 I_{s,o,dq} = \frac{1}{L_{s,o}} V_{o,dq} - \frac{R_{s,o}}{L_{s,o}} I_{s,o,dq} - \frac{1}{L_{s,o}} V_{s,dq}
\]

Each state in (1) and (2) can be split in two parts (the real component \( d \)- and the imaginary component \( q \)- of \( dq \) reference frame, respectively). Note that by setting \( * = i \) or \( * = j \) in (2) one obtains two opposite line currents \( I_{ij} \) and \( I_{ji} \), so as to have a reference current entering in each DGU. In order to guarantee that \( I_{ij}(t) = -I_{ji}(t), \forall t \geq 0 \), we introduce the following modeling assumption.

**Assumption 1.** Initial states verify \( I_{s,o,dq}(0) = -I_{s,*o,dq}(0) \). Moreover it holds \( L_{s,o} = L_{s,*o} \) and \( R_{s,o} = R_{s,*o} \).

**Remark 1.** System in (2) can be seen as an expansion of the line model one can obtain by fixing a single reference direction.

Next, we generalize model (4) to ImGs composed of \( N \) DGUs. Let \( D = \{1, \ldots, N\} \). Two DGUs \( i \) and \( j \) are neighbours if there is a transmission line connecting them and we denote with \( N_i \subset D \) the subset of neighbours of DGU \( i \). Note that, if \( j \in N_i \), then \( i \in N_j \) since the neighbouring relation is symmetric. Then, the dynamics of DGU \( i \), can be
The overall QSL-ImG model is given by
\[ \hat{x}(t) = A\hat{x}(t) + Bu(t) + Md(t) \] (5a)
\[ y(t) = Cx(t) \] (5b)
\[ z(t) = Hy(t) \] (5c)
where \( x = (x_1, \ldots, x_N) \in \mathbb{R}^{2N}, u = (u_1, \ldots, u_N) \in \mathbb{R}^{2N}, d = (d_1, \ldots, d_\delta) \in \mathbb{R}^{2N}, y = (y_1, \ldots, y_N) \in \mathbb{R}^{2N}, z = (z_1, \ldots, z_N) \in \mathbb{R}^{2N} \) and matrices \( A, B, M, C \) and \( H \) are reported in Appendix A.3 of [25].

III. PLUGIN-AND-PLAY DECENTRALIZED VOLTAGE AND FREQUENCY CONTROL

A. Decentralized control scheme with integrators

In order to track a constant set-point \( z_{\text{ref}}(t) \), when \( d(t) \) is constant, we augment the ImG model with integrators [26]. For zeroing the steady-state error, it must hold
\[ 0 = AX + Bu + M\hat{d} \] (6)
\[ z_{\text{ref}} = HCx \] (7)
where \( \bar{x} \) and \( \bar{u} \) are equilibrium states and inputs.

**Proposition 1.** Given \( z_{\text{ref}} \) and \( \bar{d} \), vectors \( \bar{x} \) and \( \bar{u} \) that satisfy (7) always exist.

**Proof.** From [26], \( \bar{x}, \bar{u} \) verifying (7) exist if and only if the following two conditions are fulfilled.

(i) The number of controlled variables is not greater than the number of control inputs.

(ii) rank(\( \Gamma \)) = 6N. This is equivalent to require that the system under control has no invariant zeros.

Condition (i) is verified since, in (9), \( u_i \) and \( z_i \) have the same size, \( \forall i \in D \). Condition (ii) can be easily proved using the definition of matrices \( A, B, C \) and \( H \) and the fact that electrical parameters are positive.

The dynamics of the integrators is (see Figure 2)
\[ \dot{z}_{\text{ref}}(t) = e_{\text{ref}}(t) - z_{\text{ref}}(t) \]
(8)
and hence, the DGU model augmented with integrators (namely \( \hat{\Sigma}_{\text{DGU}} \)) is
\[ \dot{x}_{\text{ref}}(t) = \hat{A}_{ii}\hat{x}_{\text{ref}}(t) + \hat{B}_iu_i(t) + \hat{M}_i\hat{d}_i(t) + \hat{\xi}(t) \]
(9)
\[ \hat{y}_i(t) = C_i\hat{x}_{\text{ref}}(t) \]
\[ \hat{z}_i(t) = \hat{H}_i\hat{y}_i(t) \]

where \( \hat{A}_{ii}, \hat{B}_i, \hat{M}_i \) are obtained from \( \hat{\Sigma}_{\text{DGU}} \) in (8).

**Proposition 2.** The pair \( (\hat{A}_{ii}, \hat{B}_i) \) is controllable.

**Proof.** Using the definition of controllability matrix, we have that
\[ \hat{M}_i^C = \begin{bmatrix} \hat{B}_i & \hat{A}_{ii}\hat{B}_i & \hat{A}_{ii}\hat{B}_i & \hat{A}_{ii}\hat{B}_i & \hat{A}_{ii}\hat{B}_i & \hat{A}_{ii}\hat{B}_i \end{bmatrix} =\]
(10)
Matrices \( \hat{M}_{i_{11}}^C \) and \( \hat{M}_{i_{12}}^C \) have always full rank, since all electrical parameters are positive, hence rank(\( \hat{M}^C_i \)) = 6. Therefore the pair \( (\hat{A}_{ii}, \hat{B}_i) \) is controllable.

The overall augmented system is obtained from (9) as
\[ \dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}u(t) + \hat{M}\hat{d}(t) \]
\[ \hat{y}(t) = \hat{C}\hat{x}(t) \]
\[ \hat{z}(t) = \hat{H}\hat{y}(t) \]
where \( \hat{x}, \hat{y} \) and \( \hat{d} \) collect variables \( \hat{x}_{\text{ref}}, \hat{y}_{\text{ref}} \) and \( \hat{d}_{\text{ref}} \) respectively, and matrices \( \hat{A}, \hat{B}, \hat{C}, \hat{M} \) and \( \hat{H} \) are obtained from systems (9).

B. Decentralized PnP control based on neutral interactions

In this section, we present a decentralized control approach that ensures asymptotic stability for the network of augmented DGUs \( \hat{\Sigma}_{\text{DGU}} \). Furthermore, local controllers are synthesized in a decentralized fashion allowing PnP operations. We equip each DGU \( \hat{\Sigma}_{\text{DGU}} \) with the following state-feedback controller
\[ C_i : u_i(t) = K_i\hat{y}_i(t) = K_i\hat{x}_i(t) \]
(12)
where \( K_i \in \mathbb{R}^{2 \times 6} \). Note that controllers \( C_i, i \in D \) are decentralized since the computation of \( u_i(t) \) requires the state of \( \hat{\Sigma}_{\text{DGU}} \) only. Let nominal subsystems be given by \( \hat{\Sigma}_{\text{DGU}} \) without coupling terms \( \xi_i(t) \). We design local controllers \( C_i \) such that the nominal closed-loop subsystem is asymptotically
stable. From Lyapunov theory, we can achieve this aim if there exists a symmetric matrix \( P_i \in \mathbb{R}^{6 \times 6}, P_i > 0 \) such that
\[
(\hat{A}_{ii} + \hat{B}_i K_i)^T P_i + P_i (\hat{A}_{ii} + \hat{B}_i K_i) < 0. \tag{13}
\]
Let the closed-loop QSL-ImG be given by (9) and (12). This system is asymptotically stable if the matrix \( P = \text{diag}(P_1, \ldots, P_N) \) satisfies
\[
(\hat{A} + \hat{BK})^T P + P(\hat{A} + \hat{BK}) < 0 \tag{14}
\]
where \( \hat{A}, \hat{B} \) and \( K \) collect matrices \( \hat{A}_{ij}, \hat{B}_i \) and \( K_i \), for all \( i, j \in \mathcal{D} \). Note that (13) does not imply (14), i.e. coupling terms might spoil stability of the closed-loop QSL-ImG model, as shown in [25]. In order to derive conditions such that (13) guarantees (14) we will exploit the concept of neutral interactions between subsystems (see Chapter 7 in [16]). To this purpose let us define \( \hat{A}_D = \text{diag}(\hat{A}_{ii}, \ldots, \hat{A}_{NN}) \) and \( \hat{A}_C = \hat{A} - \hat{A}_D \).

**Definition 1.** DGU interactions are neutral if the matrix \( \hat{A}_C \) can be factorized as
\[
\hat{A}_C = SP \tag{15}
\]
where \( S \) is a skew-symmetric matrix (i.e. \( S = -S^T \)).

In order to ensure asymptotic stability of the closed-loop QSL-ImG, we will exploit the following assumptions.

**Assumption 2.**

(i) The shunt capacitances at all PCCs of the microgrid are identical, i.e. \( C_{ti} = C_s, \forall i \in \mathcal{D} \).

(ii) Decentralized controllers \( C_{[i]}, \forall i \in \mathcal{D} \) are designed such that (13) holds with
\[
P_i = \begin{pmatrix}
\eta & 0 & 0 & 0 & 0 \\
0 & \eta & 0 & 0 & 0 \\
0 & 0 & \ddots & \ddots & \ddots \\
0 & 0 & \ddots & \ddots & \ddots \\
0 & 0 & \ddots & \ddots & \eta \\
0 & 0 & \ddots & \ddots & 0
\end{pmatrix} \tag{16}
\]

where \( \bullet \) denotes an arbitrary entry and \( \eta > 0 \) is a parameter common to all matrices \( P_i, \forall i \in \mathcal{D} \).

(iii) It holds \( \frac{\eta R_{ij}}{C_s Z_{ij}} \approx 0, \forall i \in \mathcal{D}, \forall j \in N_i, \) where \( Z_{ij} = |R_{ij} + j \omega_0 L_{ij}| \).

Assumption 2-(i) provides a reference ImG model for which closed-loop asymptotic stability will be shown in Proposition 3 below. However, integrators in the control loop guarantee robustness of stability [26] with respect to small deviations of capacitances \( C_{ts} \) from the common value \( C_s \). This feature will be shown through simulations in Section IV-A3. We also highlight that in electrical networks sometimes there are blocks of capacitors positioned at various PCCs that can be switched in steps so as to tune their total capacitance. In this case, Assumption 2-(i) can be directly fulfilled. As for Assumption 2-(ii) we show later that checking the existence of \( P_i \) as in (16) and \( K_i \) fulfilling (13) amounts to solving a convex optimization problem. Here, we just highlight that \( \eta > 0 \) and \( C_s > 0 \) are the only global parameters that must be known for designing local controllers.

**Remark 2.** Assumption 2-(iii) can be fulfilled in different ways. When an upper bound to all ratios \( \frac{R_{ij}}{Z_{ij}} \) (which depend upon line parameters only) is known, it is enough to set the control design parameter \( \eta \) sufficiently small. If, however, lines are mainly inductive, one has \( \frac{R_{ij}}{Z_{ij}} \approx 0 \) by construction and bigger values of \( \eta \) can be used for synthesizing local controllers.

**Proposition 3.** Let Assumption 2 holds. Then, DGU interactions are neutral and the overall closed-loop QSL-ImG is asymptotically stable.

**Proof.** We have to prove that (14) holds, i.e.
\[
(\hat{A}_D + \hat{BK})^T P + P(\hat{A}_D + \hat{BK}) + \hat{A}_C^T P + P\hat{A}_C < 0. \tag{17}
\]
Note that term (a) is a block diagonal matrix that collects on the diagonal all left hand sides of (13). Hence term (a) is a negative definite matrix. Next we prove that term (b) is zero. Considering the terms \( P_j \hat{A}_{ij} \) and using Assumption 2-(iii), we obtain
\[
P_j \hat{A}_{ij} = P_j \hat{A}_{ij} ≈ \begin{pmatrix}
\eta R_{ij} & 0 & 0 & 0 & 0 & 0 \\
0 & \eta R_{ij} & 0 & 0 & 0 & 0 \\
0 & 0 & \eta R_{ij} & 0 & 0 & 0 \\
0 & 0 & 0 & \eta R_{ij} & 0 & 0 \\
0 & 0 & 0 & 0 & \eta R_{ij} & 0 \\
0 & 0 & 0 & 0 & 0 & \eta R_{ij}
\end{pmatrix}
\]

where \( X_{ij} = \omega_0 L_{ij} \). Hence term (b) in (17) can be approximated using blocks given in (18). In the following, with a little abuse of notation, we consider coupling terms \( A_{ij} \) with zero elements on the diagonal. In this case, there always exists a skew-symmetric matrix \( S_{ij} \) such that \( \hat{A}_{ij} = S_{ij} P_j \), e.g. \( S_{ij} = \frac{1}{\eta} \hat{A}_{ij} \). Hence matrix \( S \) composed of blocks \( S_{ij} \) is skew-symmetric and such that \( \hat{A}_C = SP \). Since \( \hat{A}_C^T = -PS \), for term (b) we have
\[
(b) = \hat{A}_C^T P + P\hat{A}_C \\
= PS^T P + PSP \\
= -PSP + PSP = 0
\]

We have then shown that inequality (17) holds. \( \square \)

The main problem that still has to be solved for designing local controller \( C_{[i]} \) is the following one.

**Problem 1.** Compute a matrix \( K_i \) such that the nominal closed-loop subsystem is asymptotically stable and Assumption 2-(ii) is verified, i.e. (13) holds for a matrix \( P_i \) structured as in (16).
Consider the following optimization problem

\[
\begin{align*}
\mathcal{O}: \min & \quad \alpha_{11}\gamma_i + \alpha_{12}\beta_i + \alpha_{13}\delta_i \\
Y_i &= \begin{bmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} \\
\alpha_{12} & \alpha_{22} & \alpha_{23} \\
\alpha_{13} & \alpha_{23} & \alpha_{33}
\end{bmatrix} > 0 \quad (19a)
\end{align*}
\]

\[
\begin{align*}
Y_i\hat{A}^T_i + G_i^T\hat{B}^T_i + \hat{A}_iY_i + \hat{B}_iG_i - Y_i\gamma_iI \leq 0 \quad (19b)
\end{align*}
\]

where \(\alpha_{11}, \alpha_{12}\) and \(\alpha_{13}\) represent positive weights and \(\cdot\) are arbitrary entries. All constraints in (19) are Linear Matrix Inequalities (LMI) and therefore the optimization problem is convex and it can be efficiently solved with state-of-art LMI solvers [27].

**Lemma 1.** Problem \(\mathcal{O}\) is feasible if and only if Problem 1 has a solution. Moreover, \(K_i = G_iY_i^{-1}, P_i = Y_i^{-1}\) and \(||K_i||_2 < \sqrt{\beta}\delta_i\).

**Proof.** Inequality (13) is equivalent to the existence of \(\gamma_i > 0\) such that

\[
(\hat{A} + \hat{B}K_i)^TP_i + P_i(\hat{A} + \hat{B}K_i) + \gamma_i^{-1}I \leq 0
\]

where \(P_i\) is defined in (16). Using the Schur complement, we can rewrite (20) as

\[
\begin{bmatrix}
(\hat{A} + \hat{B}K_i)^TP_i + P_i(\hat{A} + \hat{B}K_i) & I \\
I & -\gamma_iI
\end{bmatrix} \leq 0
\]

This inequality is nonlinear in \(P_i\) and \(K_i\). As in [27], we introduce new matrices

\[
Y_i = P_i^{-1}, \quad G_i = K_iY_i.
\]

Note that \(Y_i\) has the same structure of \(P_i\). By pre- and post-multiplying (21) with \([Y_i^T \ 0 \ I]^{-1}\) and using (22) we obtain

\[
\begin{align*}
Y_i\hat{A}^T_i + G_i^T\hat{B}^T_i + \hat{A}_iY_i + \hat{B}_iG_i - Y_i\gamma_iI \leq 0 \quad (23)
\end{align*}
\]

Note that constraint (19a) guarantees that \(P_i\) has the structure prescribed by Assumption 2(ii). Moreover, the nominal closed-loop subsystem is guaranteed by (19b). In order to prevent \(||K_i||_2\) from becoming too large we add the bounds \(||G_i||_2 < \sqrt{\beta}\delta_i\) and \(||Y_i^{-1}||_2 < \delta_i\) that, via Schur complement, correspond to constraints (19c) and (19d). These bound imply \(||K_i||_2 < \sqrt{\beta}\delta_i\) and then affect the magnitude of control variables.

C. PnP operations

In this section, we discuss the operations for updating the controllers when DGUs are added to or removed from an ImG. The goal is to preserve stability of the new closed-loop system. As a starting point, we consider a microgrid composed by subsystems \(\Sigma_{DGU}^i, i \in D\) equipped with local controllers \(C_i\) and compensators \(\hat{C}_i\) and \(\hat{N}_i\) in as in Section 3.3. of [25].

**Algorithm 1** Design of controller \(C_i\) and compensators \(\hat{C}_i\) and \(\hat{N}_i\) for subsystem \(\Sigma_{DGU}^i\)

**Input:** DGU \(\Sigma_{DGU}^i\) as in (9)

**Output:** Controller \(C_i\) and, optionally, pre-filter \(\hat{C}_i\) and compensator \(\hat{N}_i\)

(A) Find \(K_i\) solving the LMI problem (19). If it is not feasible stop (the controller \(C_i\) cannot be designed).

**Optional steps**

(B) Design the asymptotically stable local pre-filter \(\hat{C}_i\) and compensator \(\hat{N}_i\) as in Section 3.3. of [25].

**Plugging-in operation** Consider the plug-in of a new DGU \(\Sigma_{DGU}^{jN+1}\) described by matrices, \(A_{N+1 N+1}, B_{N+1 N+1}, C_{N+1 N+1}, M_{N+1}, H_{N+1}\) and \(\{A_{N+1 j}\}_{j=1}^{N_{N+1}}\). In particular, \(N_{N+1}\) identifies the DGUs that are directly coupled to \(\Sigma_{DGU}^{jN+1}\) through transmission lines and \(\{A_{N+1 j}\}_{j=1}^{N_{N+1}}\) are the corresponding coupling terms. For designing controller \(C_{[N+1]}^i\) and compensators \(\hat{C}_{[N+1]}^i\) and \(\hat{N}_{[N+1]}^i, i \in D\), we execute Algorithm 1. We note that DGUs \(\Sigma_{DGU}^j, j \in N_{N+1}\), have the new neighbour \(\Sigma_{DGU}^{jN+1}\). Therefore, the redesign of controllers \(C_i\) and compensators \(\hat{C}_i\) and \(\hat{N}_i\), \(\forall j \in N_{N+1}\) is needed because matrices \(A_{jN+1}, j \in N_{N+1}\) change. In conclusion, the plug-in of \(\Sigma_{DGU}^{jN+1}\) is allowed only if Algorithm 1 does not stop in Step A when computing controllers \(C_i\) for all \(k \in N_{N+1}\) change. Note that, the redesign is not propagated further in the network, i.e. asymptotic stability of the new overall closed-loop QSL-ImG model is ensured even without changing controllers \(C_i\) of DGUs \(\Sigma_{DGU}^j, j \in N_{N+1}\).

**Unplugging operation** We consider the unplugging of DGU \(\Sigma_{DGU}^k, k \in D\). Matrix \(A_{jj}\) of each \(\Sigma_{DGU}^j, j \in N_{N+1}\) changes due to the disconnection of \(\Sigma_{DGU}^{jN+1}\) from the network. For this reason, for each \(j \in N_k\), the redesign through Algorithm 1 of controllers \(C_j\) and compensators \(\hat{C}_j\) and \(\hat{N}_j\), \(j \in N_{N+1}\), is needed and unplugging of \(\Sigma_{DGU}^{jN+1}\) is allowed only if all these
operations can be successfully terminated. As for the plugging-in operation, the redesign of local controllers $C_{[j]}$, $j \notin N_k$ is not required.

**Remark 3.** Existing contributions on decentralized control for ImG fall in two main categories. The first one comprises centralized design procedures where the use of the whole ImG model allows one to guarantee voltage and frequency stability [21], [22], [12]. The second one embraces decentralized design approaches, often based on droop control, where tuning the parameters of local regulators does not require any piece of global information about the ImG model [7], [18], [3], [28]. In this case, however, stability is seldom guaranteed. Our control design algorithm bridges the gap between the above categories, meaning that it is decentralized but, at the same time, capable to provide closed-loop stability. We also highlight that all decentralized design approaches falling in the second category allow for PnP operations. However they do not guarantee stability is preserved when a new DGU is plugged in or out. As regards the first category of contributions, all control architectures that require a centralized design do not allow for PnP operations. Indeed, when a new DGU is added, the execution of centralized design procedure can modify existing controllers of all other DGUs.

### IV. Simulation results

In this section, we study performance brought about by PnP controllers described in Section III by using the ImG in Figure 1 with two DGUs (Scenario 1) and an ImG with 10 DGUs (Scenario 2). Parameters values for all DGUs are given in Appendix C of [25]. We highlight that they are comparable to those used in [20], [18] and [23]. Simulations have been conducted in MatLab/Simulink using the SimPowerSystem Toolbox and the PnPMPMC-toolbox [29].

#### A. Scenario 1

For the sake of simplicity, we set $i = 1$ and $j = 2$ for the ImG in Figure 1. Controllers for DGUs 1 and 2 have been designed running Algorithm 1 with $i = 1$ and $i = 2$.

1) **Voltage tracking for DGU 1:** In the first test, we assess the performance in tracking step changes in the $dq$ voltage reference at $PCC_1$. For each DGU, we use an RL parallel load with constant parameters $R = 76$ $\Omega$ and $L = 111.9$ mH. The $d$ and $q$ components of the voltage at $PCC_1$ are initially set at 0.2 per-unit (pu) and 0.6 pu and those of $PCC_2$ are set at 0.5 pu and 0.7 pu, respectively. The reference signals of DGU 1, i.e., $V_{d,ref}$ and $V_{q,ref}$, are affected by two step changes: the $d$ component of the load voltage steps up to 0.3 pu at $t = 0.5$ s and the $q$ component steps down to 0.5 pu at $t = 1.5$ s. Figure 3 shows the dynamic responses of the two DGUs to these changes. In particular, Figures 3a and 3b show good tracking performances with small interactions between the two DGUs. Figures 3c and 3d show the instantaneous voltage at $PCC_1$ in the abc frame, during the two step changes of the reference signals. Note that the proposed decentralized control strategy ensures an excellent tracking of the references in about two cycles.

Additional simulations illustrating the voltage tracking at $PCC_2$ are provided in [25].

2) **Impact of a nonlinear load:** In this test, we study the performance of our controllers in presence of a highly nonlinear load. Voltage references are initially set to $V_{d,ref} = 0.8$ pu and $V_{q,ref} = 0.6$ pu for both DGUs. At the beginning of the simulation, we connect at $PCC_1$ and $PCC_2$ the RL load described in Section IV-A1. At $t = 0.5$ s, the load connected at $PCC_2$ is suddenly replaced by a three-phase six-pulse diode rectifier. The rectifier produces a DC output voltage that feeds a purely resistive load with $R = 120$ $\Omega$. We highlight that this is a standard test for assessing robustness of microgrid operations to nonlinearities (see, for example, Section VI.C in [23] and [19]). Simulations are shown in Figure 4. In particular, Figure 4a shows the $dq$ components of the load voltage at $PCC_2$ which confirm the good tracking performance of the controller in spite of the inclusion of the rectifier. From Figure 4b one can notice that, except for short transients, local controllers successfully regulate the output sinusoidal waveforms at the desired levels. Figure 4c provides a plot of the Total Harmonic Distortion (THD) (expressed in %) of load voltage at $PCC_2$. We note that, after the connection of the rectifier, the THD value grows. However, the average value of THD after $t = 0.5$ s is equal to 4% which is below the maximum limit (5%) recommended by IEEE standards in [19]. Considering that the rectifier input currents are highly distorted, as shown in Figure 4d, the control architecture ensures that the load is fed with high-quality voltages.

3) **Performance under unbalanced load conditions:** In this test, we investigate the performance of our controllers in presence of unbalanced loads and different capacitances at each PCC. The capacitances are $C_{11} = 0.9C_1$ and $C_{12} = 1.1C_1$, but we designed controllers assuming the common capacitance value $C_t = 62.86$ $\mu F$ for each DGU. In other words, we aim at testing robustness of our control scheme against deviations.
from Assumption 2-(i). Voltage references are initially set to
\(V_{d,ref} = 1\) pu and \(V_{q,ref} = 0\) pu for both DGUs. Moreover,
the nominal RL load described in Section IV-A1 is connected
at \(PCC_1\) and \(PCC_2\). At \(t = 0.5\) s the RL load parameters at
\(PCC_1\) are changed to the values given in Table I, so that the
load of DGU 1 becomes highly unbalanced.

<table>
<thead>
<tr>
<th>Phase</th>
<th>(R) ((\Omega))</th>
<th>(L) ((\mu)H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>76</td>
<td>111.9</td>
</tr>
<tr>
<td>b</td>
<td>228</td>
<td>123.9</td>
</tr>
<tr>
<td>c</td>
<td>456</td>
<td>111.9</td>
</tr>
</tbody>
</table>

**TABLE I: Unbalanced load parameters**

Figure 5a shows the \(d\) and \(q\) components of the load
voltage at \(PCC_1\) before and after unbalancing. We note that
tracking of the reference signals is still guaranteed in spite of
load changes. Moreover, instantaneous load voltages, shown in
Figure 5b, confirm successful regulation of the output
waveforms. Figure 5c shows the load current \(I_{L,1}\) provided by
DGU 1 in the abc frame. One can notice how the controller
induces major changes in the VSC behaviour in order to avoid
spoiling the balance of load voltage at \(PCC_1\). Moreover, this
test shows that, even if the controllers are designed considering
capacitance \(C_t\), thanks to the feedback, the integrators in the
control loop guarantee robustness of stability with respect to
small deviations of capacitances from a common reference
value. To evaluate the voltage imbalance at \(PCC_1\), we calculate
the ratio \(V_N/V_P\) (expressed in \%), where \(V_N\) and \(V_P\)
are the magnitudes of the negative- and positive-sequence
components of the voltage. The time evolution of this ratio
is represented in Figure 5d. We notice that it is always below
1\% which is less than the maximum permissible value (3\%)
defined by IEEE in [19].

**B. Scenario 2**

In this second scenario, we consider the ImG depicted in
black in Figure 6. Differently from Scenario 1, some DGUs have more than one neighbour and it is also present a loop
that further complicates voltage regulation.

For each subsystem \(\Sigma_{DGU}^{[i]}\), \(i \in D = \{1, \ldots, 10\}\),
we execute Algorithm 1 in order to design controllers \(C_{[i]}\)
and compensators \(\tilde{C}_{[i]}\) and \(N_{[i]}\) (see [25] for details). For evaluating
the PnP capabilities of our control approach, we simulate
the connection of DGU \(\Sigma_{DGU}^{[11]}\) with \(\Sigma_{DGU}^{[5]}\) and \(\Sigma_{DGU}^{[6]}\),
as shown in Figure 6.

Therefore, we have \(N_{[1]} = \{1, 6\}\). As described in Section
III-C, only subsystems \(\Sigma_{DGU}^{[j]}\), \(j \in N_{[1]}\) must update their
collectors \(C_{[j]}\) and compensators \(\tilde{C}_{[j]}\) and \(N_{[j]}\). This is done by re-executing
Algorithm 1 for each DGU \(\Sigma_{DGU}^{[j]}, j \in N_{[1]}\).

Then, we execute Algorithm 1 for synthesizing \(\tilde{C}_{[11]}\), \(\tilde{C}_{[1]}\)
and \(N_{[1]}\) for the new DGU. Since Algorithm 1 never stops in Step A, the addition of \(\tilde{\Sigma}_{DGU}^{[11]}\) is allowed and local controllers can be replaced by the new ones.

The real-time plugging-in of \(\tilde{\Sigma}_{DGU}^{[11]}\) is executed at time
\(t = 2\) s. Before this event, references for DGUs 1-10
are those described in Section 4.2.1 of [25] and \(\Sigma_{DGU}^{[11]}\) is
assumed to work isolated, tracking a reference voltage with \(dq\)
components \(V_{d,ref} = 1\) pu and \(V_{q,ref} = 0\) pu, respectively.
In order to test tracking performances after the addition
of \(\Sigma_{DGU}^{[11]}\), at time \(t = 2.3\) s we change the \(d\) component of
the voltage reference for \(\tilde{\Sigma}_{DGU}^{[11]}\) to 0.6 pu. Figure 7 shows
the \(dq\) component of the load voltages for \(\tilde{\Sigma}_{DGU}^{[11]}\) and its
neighbours \(\tilde{\Sigma}_{DGU}^{[5]}\) and \(\tilde{\Sigma}_{DGU}^{[6]}\). In particular, from Figures
7a and 7b, we note that right after the plugging-in time
\((t = 2\) s), the load voltages of \(\tilde{\Sigma}_{DGU}^{[11]}\) and \(\tilde{\Sigma}_{DGU}^{[6]}\)
deviate from the respective reference signals. However, this deviation
is immediately compensated and, after a short transient, the load voltages at $PCC_1$ and $PCC_6$ converge to their reference values. Similar remarks can be done for the new DGU $\Sigma_{[11]}^{DGU}$: as shown in Figures 7a and 7b, there is a short transient at the time of the plugging-in, that is effectively compensated by the control action. Moreover, the controller $\hat{C}_{[11]}$ and compensators $\hat{C}_{[11]}$ and $N_{[11]}$ ensure desired tracking when the reference signal $V_{d,ref}$ steps down at $t = 2.3$ s.

Next, we disconnect $\Sigma_{[2]}^{DGU}$. The set of neighbours of DGU 2 is $N_2 = \{1, 4\}$. Because of the disconnection, for DGUs $\hat{\Sigma}_{[j]}^{DGU}$, $j \in N_2$ there is a change in their local dynamics $\hat{A}_{jj}$. Then, as described in Section III-C, each subsystem $\hat{\Sigma}_{[j]}^{DGU}$, $j \in N_2$ must redesign controller $\hat{C}_{[j]}$ and compensators $\hat{C}_{[j]}$ and $N_{[j]}$. Hence, matrices $\hat{A}_{jj}$, $j \in N_2$, are updated and then Algorithm 1 is re-executed. Since Algorithm 1 never stops in Step A, the unplugging of $\Sigma_{[2]}^{DGU}$ is allowed. Finally, we simulate the unplugging operation by disconnecting DGU $\Sigma_{[2]}^{DGU}$ at time $t = 2.6$ s. As shown in Figure 8, the $dq$ components of load voltages of DGU $\Sigma_{[j]}^{DGU}$, $j \in N_2$ deviate from the respective reference signals. Thanks to the retuning of the controllers $\hat{C}_{[j]}$ and compensators $\hat{C}_{[j]}$ and $N_{[j]}$, $j \in N_2$, this deviation is immediately compensated and, after a short transient, the load voltages at $PCC_1$ and $PCC_4$ converge to their respective steady state values. Also in this case, stability of the microgrid is preserved despite the disconnection of $\Sigma_{[2]}^{DGU}$.

![Fig. 6: Scenario 2 - Scheme of the microgrid composed by 10 DGUs (in black) and plugging-in of $\Sigma_{[2]}^{DGU}$ (in red).](image)

![Fig. 7: Scenario 2 - Performance of decentralized voltage control during the plug-in operation ($t = 2$ s) and a change in the reference for DGU 11 ($t = 2.3$ s).](image)

![Fig. 8: Scenario 2 - Performance of decentralized voltage control during the unplugging operation ($t = 2.6$ s).](image)

**V. Conclusions**

In this paper, we presented a decentralized control scheme for guaranteeing voltage and frequency stability in ImGs. Differently from other decentralized controllers available in the literature (e.g. [21], [22], [3]), a key feature of our approach is that plugging-in and -out of DGUs requires to update only a limited number of local controllers. Furthermore, a global model of the ImG is not required in any design step. Numerical results in Section IV and in [25] confirm effectiveness of PnP control even for ImGs with meshed topology and
components accounting for nonlinearities commonly found in practice. Voltage and frequency control takes place at a very fast timescale where renewable sources (commonly equipped with storage devices) can be modeled as constant voltage generators. However, this approximation is no longer valid for describing the behaviour of ImGs over longer time horizons. In this case, dynamics and stochasticity of the sources plays an important role. This topic will be addressed in future research. Furthermore, local voltage controllers should be coupled with a higher control layer devoted to power flow regulation so as to orchestrate mutual help among DGUs. To this purpose, we will study if and how ideas from primary control of ImGs [3] can be reappraised in our context.

VI. ACKNOWLEDGMENT

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REFERENCES