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Neural Network Implementation of Nonlinear Receding-Horizon Control*

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The Receding Horizon (RH) *approach is an effective way to derive control algorithms for nonlinear systems with stabilising properties also in the presence of state and control constraints. However,* RH *methods imply a heavy computational burden for on-line optimisation, therefore they are not suitable for the control of 'fast' systems, for example mechanical ones, which call for the use of short sampling periods. The aim of this paper is to show through an experimental study how a Nonlinear* RH (NRH) *control law can be computed off-line, and subsequently approximated by means of a neural network, which is effectively used for the on-line implementation. The proposed design procedure is applied to synthesise a neural* NRH *controller for a seesaw equipment. The experimental results reported here demonstrate the feasibility of the method.*

Keywords: Mechanical systems; Neural network; Nonlinear control; Receding-Horizon Control

1. Introduction

In recent years, many Nonlinear Receding Horizon (NRH) control algorithms have been proposed with guaranteed local stability properties even when constraints are imposed on the evolution of the control and state variables [1–6]. In particular, the technique presented by De Nicolao et al. [5] provides exponential stability of the equilibrium under the mild assumption of stabilisability of the associated linearised system. The corresponding NRH control law is computed through the solution of a Finite Horizon (FH) optimisation problem with optimisation horizon N and terminal state penalty equal to the cost that would be incurred by applying a local stabilising linear control law thereafter. It is remarkable, however, that the linear control law is never applied in practice, but is just used to compute the terminal state penalty. Moreover, the region of attraction of the equilibrium grows with N, and tends to that of the associated Infinite Horizon (IH) optimisation problem. For this reason, one should select long horizons N in order to improve the overall performance and to enlarge the exponential stability region. On the other hand, as N increases the optimisation problem becomes more and more difficult to solve, and it is surely intractable for an on-line implementation on 'fast' applications, i.e. when the dynamics of the system under control force the use of a small sampling period.

In these cases, the procedure first suggested by Parisini and Zoppoli [3] can be followed. Specifically, one can compute off-line the optimal NRH control law $\kappa^{RH}(x)$ for a (large) set of admissible values of the initial state $x \in X$. Then, it is possible to approximate $\kappa^{RH}(x)$ with any suitable interpolation technique, for example by means of a Neural Network (NN). Finally, the approximating function so obtained is effectively implemented for on-line computations.

Although the above implementation procedure is conceptually very attractive and has been used in simulation experiments [3], to the author's knowledge, no real applications have been presented so far.

The aim of this paper is to present some experimental results obtained in the control of a seesaw apparatus with a NN approximation of the NRH

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control law presented by De Nicolao et al. [5]. To reduce the computational burden required to determine the control sequences used for the training of the NN, the NRH control law is computed for initial values of the system state which are far from the equilibrium, while the standard optimal Linear Quadratic (LQ) technique is applied in its neighbourhoods.

The paper is organised as follows. In Section 2, the state feedback NRH control law [5] is briefly summarised together with its properties. Moreover, since the state of the seesaw equipment is not completely accessible, the extension of the NRH control law to the case of output feedback is presented together with the associated stabilising results, recently presented by Magni et al. [7]. Section 3 describes the guidelines followed to derive the NN approximation of the NRH control law. In Section 4, the experimental apparatus is presented; the implemented NRH/NN control law is described, and some experimental results are reported to witness the applicability of the approach. Finally, some concluding remarks are reported in Section 5.

2. Nonlinear Receding-Horizon Control

In this section, we briefly review the results in De Nicolao et al. [5] and Magni et al. [7] on state feedback and output feedback RH control of nonlinear systems, which form the basis of all the subsequent developments.

The nonlinear discrete-time dynamic system is assumed to be described by

$$x(k+1) = f(x(k),u(k)), \quad x(t) = \bar{x} \quad k \geq t \tag{1}$$

$$y(k) = h(x(k)) \tag{2}$$

where $x \in R^n$ is the state, $y \in R^m$ is the output, and $u \in R^m$ is the input. The functions $f(\cdot,\cdot)$ and $h(\cdot)$ are C^1 functions of their arguments and $f(0,0) = 0$, $h(0) = 0$.

For the system (1), we search for a NRH control law $u = \kappa^{RH}(x)$ which regulates the state to the origin, subject to the input and state constraints

$$x(k) \in X, \quad u(k) \in U, \quad k \geq t \tag{3}$$

where X and U are closed subsets of R^n and R^m, respectively, both containing the origin as an interior point.

To derive the NRH control law, first let

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) \tag{4}$$

be the linearisation of system (1)–(2) around the equilibrium point $(x,u) = (0,0)$, i.e.

$$A = \frac{\partial f}{\partial x}(0,0), \quad B = \frac{\partial f}{\partial u}(0,0), \quad C = \frac{\partial h}{\partial x}(0)$$

Assuming that the pair (A,B) is stabilisable, well known results of linear control theory state that it is possible to find a matrix K such that the eigenvalues of $(A+BK)$ are inside the unit circle in the complex plane. Note that K is the gain matrix of a linear state feedback control law $u(k) = Kx(k)$ which stabilises the linear system (4). Hence, K can be computed by means of standard synthesis methods for linear systems, for example LQ [8] or pole-placement [9] techniques.

Now, for a given stabilising matrix K, at any time instant t let $\bar{x} = x(t)$ and minimise with respect to $u_{t,t+N-1} := [u(t)\ u(t+1)\ \ldots\ u(t+N-1)]$, $N \geq 1$, the cost function

$$J(\bar{x},u_{t,t+N-1},N,K) = \sum_{i=0}^{N-1} x'(t+i)Qx(t+i) + u'(t+i)Ru(t+i) + V_f(x(t+N),K) \tag{5}$$

subject to (1) and (3), with $Q > 0$, $R > 0$ and the terminal state penalty V_f defined as

$$V_f(\bar{x},K) = \sum_{i=0}^{\infty} x'(t+i)(Q+K'RK)x(t+i)$$

where $x(t+i)$, $i \geq 0$ satisfies (1) with $u(k) = Kx(k)$.

The optimal control sequence $u^o_{t,t+N-1}$ solving the above optimisation problem is termed *admissible* if, when applied to system (1)

$$x(k) \in X,\ u(k) \in U,\ t \leq k < t+N$$
$$x(t+N) \in X(K)$$

where $X(K)$ stands for the exponential stability region (see [1] and [5]) of the nonlinear closed-loop system composed by the nonlinear system (1) and the linear control law $u(k) = Kx(k)$. In other words, $\bar{x} \in X(K)$ implies the fulfillment of the constraints (3), i.e. $x(k) \in X$, $Kx(k) \in U, k \geq t$.

Finally, the state-feedback NRH control law $u = \kappa^{RH}(x)$ is obtained by applying at any time instant t the control $u(t) = u^o(\bar{x})$ where $u^o(\bar{x})$ is the first column of $u^o_{t,t+N-1}$. Letting $X_0(N,K)$ be the set of states $\bar{x}$ such that any admissible control sequence $u^o_{t,t+N-1}$ exists, the following result holds:

Theorem 1 [5] Assume that (A,B) is stabilisable and let K be such that the eigenvalues of $(A+BK)$ are inside the unit circle in the complex plane. Then, if the NRH control law $u = \kappa^{RH}(x)$ is applied to the nonlinear system (1), the origin is an exponentially stable equilibrium point of the resulting closed-loop system having $X_0(N,K)$ as exponential stability region.

A very practical procedure for stabilising the nonlinear system (1) is to design first a linear control law $u = \kappa_L^{IH}(x) = Kx$ by minimising an IH performance index subject to the linearised state dynamics (4). In this respect, well-established tools are available for the tuning of the weighting matrices Q and R in a standard LQ control problem so as to achieve the desired specifications for the linearised closed-loop system $x(k+1) = (A+BK)x(k)$. Then, the same Q and R are used to implement the nonlinear RH controller. Under regularity assumptions, as $\|x\| \to 0$, it turns out that $\kappa^{RH}(x) \to \kappa_L^{IH}(x) = Kx$. Moreover, $(\partial \kappa^{RH}(x)/\partial x\|_{x=0} = K$ so that the NRH control law can be regarded as a consistent nonlinear extension of the linear control law $u = Kx$.

In this procedure, once Q and R have been selected, the only free parameter is the optimisation horizon N, which can be tuned to trade computational complexity (which grows with N) for performance ($\kappa^{RH}(x) \to \kappa^{IH}(x)$ as $N \to \infty$, where $\kappa^{IH}(x)$ is the unknown optimal control law for the IH nonlinear problem). Furthermore, as N grows, the stability region enlarges and $X_0(N,K) \to X^{IH}$, where X^{IH} is the region of attraction of the optimal IH nonlinear control law $\kappa^{IH}(x)$. It can also be proven that $X_0(N+1,K) \supseteq X_0(N,K) \supseteq X(K)$ $\forall N > 0$ [5].

The NRH state-feedback control law previously introduced assumes the knowledge of the system state x at any time instant t. In many practical cases, for example in the application described in Section 4, only the system outputs are measured. Then, the state-feedback control law must be combined with a suitable state observer producing at any time instant t the estimation $\hat{x}(t)$ of the state vector $x(t)$ from the measures of the system inputs and outputs. Finally, the truly implemented NRH 'output feedback type' control law is computed on the estimated state, that is $u = \kappa^{RH}(\hat{x})$. However, this procedure raises an important theoretical issue: to what extent the stabilising properties of the state feedback control law (see Theorem 1) still hold in the output feedback case? This problem has been analysed in depth by Magni et al. [7], where it has been shown that, under mild observability properties on the original nonlinear system (1)–(2), combining the NRH state-feedback control law with popular observer methods, such as the Kalman filter, the asymptotic (or exponential) stability of the equilibrium is still guaranteed.

3. Neural Network Implementation of the NRH Control Law

The main drawback of the RH approach is the necessity to solve a nonlinear optimisation problem on-line. This is possible for 'slow' systems such as the one considered by Magni et al. [10] or chemical and petrochemical plants where NRH control is already widely industrially applied. However, notwithstanding the improvements of the hardware technology, a direct on-line implementation of NRH control may be still quite impossible for 'fast' systems.

To apply the NRH approach also when a short sampling interval must be used, one can consider to solve the FH optimisation problem off-line for many different initial states x and to store the computed sequences of control variables, i.e. the 'realisations' of $\kappa^{RH}(x)$, in the computer's memory. Then, in the real application, one has to select or interpolate the computed control sequence that best fits with the current state of the plant. Clearly, this strategy has the disadvantage that an excessive amount of computer memory may be required to store the closed-loop control law. Moreover, some interpolating procedure must be implemented in practice to effectively compute the control variable.

An interesting way to solve these implementation problems is to resort to a functional approximation $\tilde{\kappa}^{RH}(x)$ of the optimal control law $\kappa^{RH}(x)$ [3]. More specifically, we search for a function $\tilde{\kappa}^{RH}(x,w)$, with a given structure, which depends upon a vector of parameters w. The values of w must be optimised with respect to the approximation error

$$E(w) = \sum_{i=0}^{N_c} \|\kappa^{RH}(x(i)) - \tilde{\kappa}^{RH}(x(i),w)\|$$

where $x(i)$, $i = 1, \ldots, N_c$, are the states of the sequences which have been computed off-line, and that form the training set.

Among the many different approximation strategies nowadays available, in this work we have concentrated on multilayer feedforward neural networks [11]. In particular, we assume that the approximating neural function $\tilde{\kappa}^{RH}(x,w)$ contains only one hidden layer composed of ν neural units (perceptron) with a sigmoidal activation function

$$f_s(y) = (1 + e^{-\beta y})^{-1}$$

and that the output layer is composed of linear activation units. It is well known that continuous functions can be approximated to any degree of accuracy on a given compact set by feedforward neural networks based on sigmoidal functions, provided that the number of perceptrons ν is sufficiently large. However, the choice *a priori* of the number of perceptrons to use is an open problem of neural network approximation theory. In Parisini and Zoppoli [3], a theoretical study of the approximating properties of the receding-horizon neural regulator is reported.

4. NRH Control of a Seesaw

The design procedure described in the previous sections has been followed for the synthesis of a neural controller for a seesaw. The apparatus, schematically shown in Fig. 1, consists of two long arms hinged onto a triangular support. The axis is coupled to a potentiometer which allows one to measure the seesaw angle. A cart slides on a ground strainless steel shaft. The cart is equipped with a motor and a potentiometer. These are coupled to a rack and pinion mechanism to input the driving force to the system and to measure cart position respectively. The control objective is to design an output feedback control law that controls the position of the cart to maintain the seesaw in the horizontal position.

4.1. Nonlinear Continuous Time Model

Denoting by p and v the cart position and velocity and by θ and φ the angle position and velocity (see Fig. 1), the nonlinear model of the plant is given by the following differential equations:

$$\dot{p} = v$$

$$\dot{v} = \left[\frac{mp^2+J+mh^2}{mp^2+J}\right]\left(\frac{F}{m} + \varphi^2 p\right) + - \frac{g}{mp^2+J}\left[(Mhc-mp^2-J)\sin(\theta) + (mhp\cos(\theta))\right] + \frac{2mh\varphi v p}{mp^2+J}$$

$$\dot{\theta} = \varphi$$

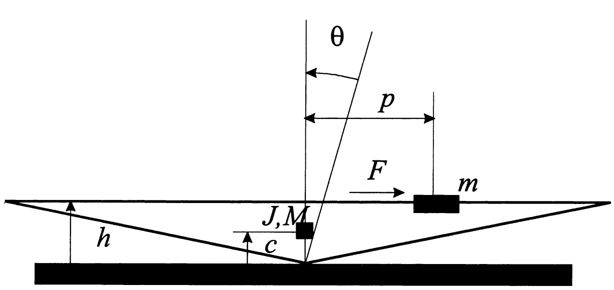

Fig. 1. Mechanical scheme of the seesaw.

$$\dot{\varphi} = \left[-\frac{mh}{mp^2+J}\right]\left(\frac{F}{m}+\varphi^2 p\right) + \frac{1}{mp^2+J}\left[Mgc\sin(\theta) + mgp\cos(\theta)\right] - \frac{2m\varphi v p}{mp^2+J} \tag{6}$$

where M and m are the seesaw and cart masses, J is the moment of inertia, h is the height of the track from pivot point, c is the centre of mass seesaw (height from pivot point), g is the acceleration due to gravity and F is the force applied to the cart. The force is from a DC motor coupled to a track via a rack and pinion mechanism. It depends upon the input voltage u in the following way:

$$F = \frac{k_g K_m}{rR_a} u - \left(\frac{k_g K_m}{r}\right)^2 \frac{1}{R_a} v \tag{7}$$

where R_a is the armature resistance, k_g is the built-in gear ratio of motor, r is the radius of output pinion and K_m is the torque constant. In the experimental apparatus used in this study, the values of the parameters are: m = 0.455 Kg, M = 3.3 Kg, h = 0.1397 m, c = 0.058 m, J = 0.427 Kgm^2, g = 9.81 m/s^2 with k_g = 3.7, K_m = 0.00767 Nm/A, r = 0.0064 m, R_a = 2.6 Ω. The maximum allowed value of θ is of about $\pm 13°$ for physical constraints.

Note that model (6) does not account for the presence of friction, which can be viewed in the control synthesis phase as an unmodelled dynamics. The robustness of the controller will also be tasted against this uncertainty.

In the (unstable) equilibrium $x = [p \;\; v \;\; \theta \;\; \varphi]' = [0 \;\; 0 \;\; 0 \;\; 0]'$, the seesaw is horizontal and the cart is located at the centre of the triangular support. The system is assumed to be controlled with a digital controller with sampling period equal to 5 ms.

4.2. NRH Control Law and Neural Approximation

According to the procedure outlined in Section 2, the linearlisation of model (6)–(7) around the origin has first been computed and discretised. The obtained discrete-time linear model is defined by the matrices A and B reported in the Appendix, and is able to describe with accuracy the seesaw dynamics for angle positions θ roughly ranging in the interval $(-8°, 8°)$. On the contrary for $|\theta| \geq 8°$ the nonlinear behaviour dominates.

With reference to the discrete-time linearised system (4), the stabilising gain K (see again the Appendix) of the linear control law $u = Kx$ has been determined by means of the LQ method [8] with state and control weighting matrices Q and R given by

$$Q = diag\{3000,0.1,3000,0.1\}, \quad R = 2 \tag{8}$$

Note that, since the seesaw behaviour is almost linear for $|\theta^0|<8°$, the control value u computed with the linear control law is almost equal to the one which could be determined through the minimisation of the performance index (5) referred to the nonlinear system (6)–(7), i.e. $u = Kx \simeq \kappa^{RH}(x)$. For this reason, starting from different initial conditions $x_0 = [0\ 0\ \theta^0\ 0]'$, $|\theta^0| < 8°$, the control law $u = Kx$ has been used to generate with a negligible computational effort the control sequences to be subsequently used for the training of the neural net.

To enlarge the stability region and to improve the control performance, the NRH control algorithm of Section 2 has been used to compute off-line the optimal control sequences corresponding to various initial conditions $x_0 = [0\ 0\ \theta^0\ 0]'$, with $8° \leq |\theta^0| \leq 13°$. The optimisation horizon $N = 10$ has been used together with the Q and R matrices again given by Eq. (8). As discussed in Section 3, the computed sequences are 'realisations' of the truly optimal NRH control law $u = \kappa^{RH}(x)$, and have been subsequently used in the training of the approximating net. Observe that the solution of the optimisation problem requires a significant computational burden.

Finally, a multilayer feedforward neural net with 30 perceptrons and sigmoidal activation function with $\beta = 1$ has been used to approximate the state-feedback NRH control law. As already stated, the training-set has been composed both of the sequences computed with the linear control law ($|\theta^0| < 8°$) and of those determined through the optimisation phase. In so doing, a smooth passage from the nonlinear control law to the linear LQ one has been guaranteed by the training process itself.

4.3. Output Feedback Control Law

In the seesaw experimental apparatus only the cart and the angle positions are directly measured and coincide with the outputs of the system, while the cart and angle velocities must be reconstructed with an observer. In particular, a standard Kalman filter [12] has been derived for the discrete-time linearised system, and used for on-line closed-loop control. This filter has been designed assuming that two Gaussian and mutually independent white noises $\xi \sim WGN(0,Q^*)$, $Q^* = diag[0.0001\ 0\ 0\ 0]$ and $\psi \sim WGN(0,R^*)$, $R^* = diag[0.0004\ 0.0004]$ act on the state and output vectors, respectively. Correspondingly, the filter gain L reported in the Appendix has been computed.

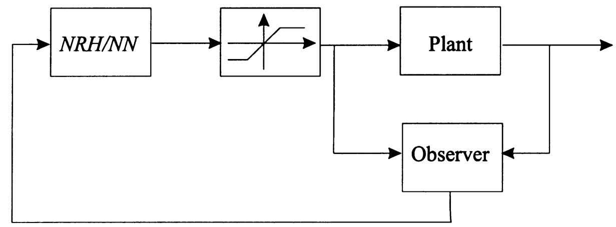

Fig. 2. Control scheme.

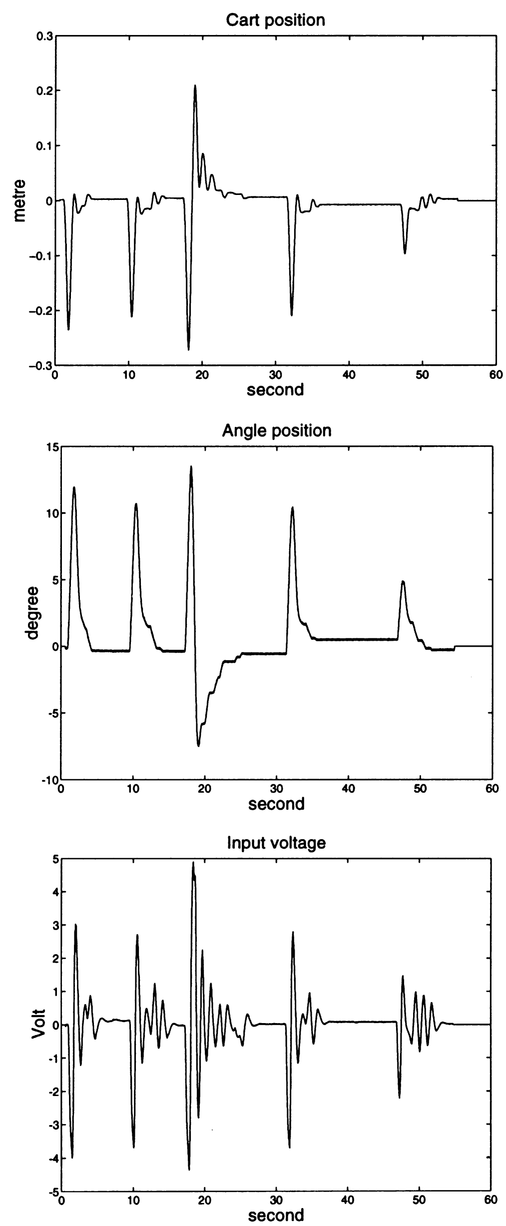

Fig. 3. Experimental results.

In summary, the scheme used for on-line control is shown in Fig. 2. It is composed by the plant, the Kalman observer, the NRH neural network (NRH/NN) state-feedback control law and a saturation block. This control scheme has been implemented on a PC-133 MHz using a C-language program and a commercial data acquisition system.

4.4. Experimental Results

To test the performance of the NRH/NN control law, some perturbations have been imposed to the seesaw. In particular, 'impulse-type' forces have been provided to one extremum of the apparatus, so that the task of the control system has been to bring back the seesaw in the horizontal position.

Some of the results achieved are presented in Fig. 3, where the transients of the cart position, of the angle position and of the input voltage are reported. Concerning these figures, two facts have to be noted. First, the NRH/NN control law guarantees good results also for perturbations of the angle position θ greater than 8^0 where the nonlinear control synthesis procedure is effective. Second, there is a small steady-state error both in the cart and in the angle positions. Correspondingly, in the steady-state the input voltage is different from zero. This is caused by the presence of friction forces that for small values of the input u prevent the cart from moving. To eliminate these errors, one should include suitable integral action into the feedback loop.

5. Conclusion

The results reported in this paper clearly illustrate that the Receding Horizon approach is a practical way to solve nonlinear control problems also for 'fast' systems. In these cases, the true nonlinear control law must be computed off-line first. Then, it can be approximated through the use of nowadays standard tools, such as Neural Nets. The approach followed here has been used also to control a laboratory inverted pendulum with excellent results [13].

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Appendix

Locally stabilising control law

$$K = [92.63 \quad 8.67 \quad 97.53 \quad 36.50]$$

designed by solving an LQ problem with the Q and R matrices given by Eq. (8) based on the discrete-time linearised system

$$x(k+1) = Ax(k) + Bu(k)$$

with

$$A = \begin{bmatrix} 1.0000 & 0.0048 & 0.0001 & 0.0000 \\ -0.0070 & 0.9187 & 0.0441 & 0.0001 \\ 0.0001 & 0.0000 & 1.0001 & 0.0050 \\ 0.0522 & 0.0120 & 0.0223 & 1.0001 \end{bmatrix};$$

$$B = \begin{bmatrix} 0.0000 \\ 0.0185 \\ 0.0000 \\ -0.0027 \end{bmatrix}$$

Kalman Filter gain

$$L = \begin{bmatrix} 0.3904 & 0.0004 \\ -0.0037 & 0.0130 \\ 0.0004 & 0.0288 \\ 0.0320 & 0.0843 \end{bmatrix}$$

derived from the following noisy discrete-time linearised systems:

$$x(k+1) = Ax(k) + Bu(k) + \xi(k) \quad x(0) = \bar{x}$$

$$y(k) = Cx(k) + \psi(k)$$

where

$$C = [1\ 0\ 1\ 0]$$

and $\bar{x}$, $\xi(k)$ and $\psi(k)$ are assumed jointly Gaussian and mutually independent. Furthermore, $\bar{x} \sim N(0,I)$, $\xi(k) \sim WN(0,Q^*)$, $\psi(k) \sim WN(0,R^*)$, with $Q^* = diag[0.0001\ 0\ 0\ 0]$ and $R^* = diag[0.0004\ 0.0004]$.