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Stabilizing decentralized model predictive control of nonlinear systems☆

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Abstract

This note presents a stabilizing decentralized model predictive control (*MPC*) algorithm for nonlinear discrete time systems. No information is assumed to be exchanged between local control laws. The stability proof relies on the inclusion of a contractive constraint in the formulation of the *MPC* problem.

Keywords: Nonlinear model predictive control; Decentralized control; Discrete time systems; Stabilization

1. Introduction

The development of synthesis methods for decentralized control schemes has received a great attention for a long time, and many results are nowadays available, see e.g. Siljak (1991) and the references reported therein, Ioannou (1986), Han and Chen (1995), and Jiang (2002), just to cite a few. In fact, a distributed control structure is often the most appropriate one in many different fields, such as in the power industry, in aerospace and chemical applications or in the manufacturing industry.

At the same time, the last two decades have seen the widespread diffusion of model predictive control (*MPC*) techniques, which are now recognized as the most useful approach to deal with the control problems typical of the process industry. Indeed, with *MPC* it is possible to formulate the control problem as an optimization one, where many different (and possibly conflicting) goals are easily formalized and state and control constraints can be included. Also for *MPC*, many results are nowadays available concerning stability and robustness, see e.g. Mayne, Rawlings, Rao, and Scokaert (2000), so that it can now be seen as a well assessed methodology.

In view of the above considerations, it is then natural to look for *MPC* algorithms to be implemented according to a decentralized structure. Indeed, the possibility to use *MPC* in a decentralized fashion can also have the advantage to reduce an original, large size, optimization problem into a number of smaller and easily tractable ones. Decentralized *MPC* methods have already been studied in Dunbar and Murray (2004), and Camponogara, Jia, Krough, and Talukdar (2002) and in a number of papers quoted there. In particular, in Camponogara et al. (2002), the system under control has been assumed to be composed by a number of linear discrete-time subsystems, and different information structures have been considered, all of them guaranteeing the possibility to exchange some kind of information between the distributed controllers.

Conversely, in this paper, a stabilizing decentralized *MPC* algorithm is derived under the main assumptions that the overall system under control is nonlinear, discrete-time and no information can be exchanged between local control laws, i.e. a fully decentralized information structure is considered. The proposed method deeply relies on the *MPC* approach presented in de Oliveira Kothare and Morari (2000), where the closed-loop stability property is achieved through the inclusion in the optimization problem of a contractive constraint. With respect to other methods often adopted in *MPC* to achieve stability, see

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e.g. Mayne et al. (2000) or Magni and Scattolini (2004), the use of a contractive control law is preferred here since it does not require the knowledge of an auxiliary stabilizing control law, which could be difficult to derive in view of the distributed nature of the problem.

2. Problem statement

Let the system under control be composed by the interconnection of N local subsystems described by the following nonlinear, discrete-time models:

$$x_i^p(k+1) = f_i(x_i^p(k), u_i(k)) + g_i(x^p(k)) + d_i(k),$$
$$x_i^p(0) = x_{i0}^p, \quad i = 1, \ldots, N, \tag{1}$$

where $x_i^p \in R^{v_i}$ is the state of the ith subsystem, $d_i \in R^{r_i}$ is the disturbance, while u_i is the control which is restricted to fulfill the following constraint:

$$u_i(k) \in U_i, \quad k \geqslant 0, \tag{2}$$

where U_i is a compact subset of R^{m_i} containing the origin as an interior point.

In (1), the mutual influence of the N subsystems is described by the functions g_i, which depend on the overall state

$$x^p(k) = \left[x_1^{p\prime}(k) \;\; x_2^{p\prime}(k) \;\; \ldots \;\; x_N^{p\prime}(k) \right]' \in R^v, \quad v = \sum_{i=1}^{N} v_i.$$

Define also the overall disturbance vector

$$d(k) = \left[d_1'(k) \;\; d_2'(k) \;\; \ldots \;\; d_N'(k) \right]' \in R^r, \quad r = \sum_{i=1}^{N} r_i.$$

Concerning subsystems (1), the following assumptions are introduced:

Assumption 1. The functions f_i, $i = 1, \ldots, N$, are C^1 functions of their arguments, such that $f_i(0, 0) = 0$ and the following Lipschitz condition is verified:

$$|f_i(\xi_i, u_i) - f_i(\zeta_i, u_i)| \leqslant L_{f_i} |\xi_i - \zeta_i|,$$
$$i = 1, \ldots, N, \quad \xi_i, \zeta_i \in R^{v_i}.$$

Assumption 2. There exist positive Lipschitz constants L_{ij}, $i, j \in [1, 2, \ldots, N]$, such that

$$|g_i(x^p)| \leqslant \sum_{j=1}^{N} L_{ij} |x_j^p|, \quad i = 1, \ldots, N.$$

As for the disturbances d_i, letting $N_{\rm p}$ be a given positive integer, henceforth called the "prediction horizon", it is assumed that the following assumption is fulfilled. □

Assumption 3. The disturbances d_i, $i = 1, \ldots, N$, are asymptotically decaying and bounded, that is,

$$d_i(k) \in B_{\bar{\rho}_d} := \{d_i \in R^{r_i} : |d_i| \leqslant \bar{\rho}_d \in [0, \infty)\}, \quad k \in Z_+,$$

where Z_+ is the set of nonnegative integers.

The problem here considered can now be formally stated as the one of finding a set of N local control laws

$$u_i(k) = \kappa_i(x_i^p(k)), \quad i = 1, \ldots, N, \tag{3}$$

such that, under Assumptions 1–4, the origin of the overall system composed by the N subsystems (1) and control laws (3) is an asymptotically stable fixed point defined, according to Scokaert, Rawlings, and Meadows (1997), as follows: □

Definition 1. The origin is an asymptotically stable fixed point of the perturbed system (1), (3) if:

(i) there exist strictly positive constants $\tilde{\rho}_i$, ρ_i^0 and $\bar{\rho}_d$, $i = 1, \ldots, N$, such that, if $x_{i0}^p \in B_{\rho_i^0}$, $i = 1, \ldots, N$, and $d_i(k) \in B_{\bar{\rho}_d}$, $i = 1, \ldots, N$, for all $k \geqslant 0$, then the solution of the ith perturbed system (1), (3), $i = 1, \ldots, N$, remains in a ball $B_{\tilde{\rho}_i}$ for all $k \geqslant 0$;
(ii) if $x_{i0}^p \in B_{\rho_i^0}$, $i = 1, \ldots, N$, and $d_i(k) \to 0$ as $k \to \infty$, $i = 1, \ldots, N$, then the solution of the ith perturbed system (1), (3) converges asymptotically to the origin.

3. Decentralized state-feedback MPC

The contractive *MPC* algorithm can now be formally stated as in de Oliveira Kothare and Morari (2000). To this end, letting

$$\bar{u}_i(t) = [u_i(t) \;\; u_i(t+1) \;\; \ldots \;\; u_i(t + N_{\rm p} - 1)],$$

the ith, $i = 1, \ldots, N$, decentralized control law (3) is obtained by (locally) minimizing at any time instant t and with respect to $\bar{u}_i(t)$ the following performance index:

$$J_i(x_i(t), N_{\rm p}) = \sum_{j=t}^{t+N_{\rm p}} |x_i(j)|^2_{Q_i} + |u_i(j)|^2_{R_i} \tag{4}$$

subject to constraints (2) and

$$x_i(k+1) = f_i(x_i(k), u_i(k)), \quad x_i(t) = x_i^p(t), \quad k \geqslant t, \tag{5}$$

$$|\bar{x}_i^t(nN_{\rm p} + N_{\rm p})| < \alpha_i |x_i^p(nN_{\rm p})|, \quad \alpha_i \in [0, 1), \tag{6}$$

where $n = \max_{\lambda \in Z_+} \lambda N_{\rm p} \leqslant t$,

$$\bar{x}_i^t(k+1) = f_i(\bar{x}_i^t(k), u_i(k)), \quad k \geqslant t, \tag{7}$$

$$\bar{x}_i^t(t) := \begin{cases} x_i^p(t) & \text{if } t = nN_{\rm p}, \\ \bar{x}_i^{t-1}(t) & \text{if } t \neq nN_{\rm p}. \end{cases} \tag{8}$$

In the definition of J_i, the positive integer $N_{\rm p}$ is the prediction horizon assumed for simplicity equal for any subsystem, while Q_i and R_i are positive definite matrices. Note that:

(i) the minimization is performed with respect to the nominal model coinciding with (1) when the system is decoupled and the disturbance is null;
(ii) the contractive constraint (6), which is crucial for the closed-loop stability, is modified every $N_{\rm p}$ time steps.

Thus constraint is imposed on the trajectory $\bar{x}_i^t$ defined through (7) and (8).

According to the receding horizon approach, for the ith subsystem the state-feedback *MPC* control law is derived by solving at any sampling time t the optimization problem (4)–(8) and by applying the control signal $u_i(t)$. In so doing, one implicitly defines the decentralized state-feedback control laws

$$u_i = \kappa_i^{\mathrm{RH}}(x_i^p), \quad i = 1, \ldots, N. \tag{9}$$

In order to derive the main stability result, the following assumption is introduced.

Assumption 4. For subsystems (1), (9), $i = 1, \ldots, N$, with $d_i(k) = 0$ and $g_i(x^p(k)) = 0$, there exist $\beta_i \in (0, \infty)$ so that

$$x_i^p(k) \in B_{\beta_i |x_i^p(nN_\mathrm{p})|}, \quad k \in [nN_\mathrm{p}, (n+1)N_\mathrm{p}), \quad n \in Z_+.$$

If (9) are Lipschitz state-feedback functions of x_i^p, the values of β_i, $i = 1, \ldots, N$, can be easily computed. □

Lemma 1. *Under Assumptions* 1, 2, 4 *let* $\bar{\rho}_i$ *be such that* $\forall x_i^p(nN_\mathrm{p}) \in B_{\bar{\rho}_i}$ *there exists a feasible solution of the optimization problem* (4)–(8), $\forall i = 1, \ldots, N$. *Then, defining* $\forall k > 0$,

$$D(nN_\mathrm{p}, k) = [|d(nN_\mathrm{p})| \; |d(nN_\mathrm{p}+1)| \; \ldots \; |d(nN_\mathrm{p}+k-1)|]' \in R^k$$

and

$$X^p(nN_\mathrm{p}) = [|x_1^p(nN_\mathrm{p})| \; |x_2^p(nN_\mathrm{p})| \; \ldots \; |x_N^p(nN_\mathrm{p})|]' \in R^N,$$

there exist computable functions

$$\gamma_i(X^p(nN_\mathrm{p}), D(nN_\mathrm{p}, k), k), \quad i = 1, \ldots, N,$$

such that for the closed-loop systems (1), (9) *and* (7), (9), $\forall k = 1, \ldots, N_\mathrm{p}$, $\forall i = 1, \ldots, N$, *the following relation holds*:

$$|x_i^p(nN_\mathrm{p}+k) - \bar{x}_i(nN_\mathrm{p}+k)| \leqslant \gamma_i(X^p(nN_\mathrm{p}), D(nN_\mathrm{p}, k), k).$$

Defining

$$\bar{\rho} = [\bar{\rho}_1 \; \bar{\rho}_2 \; \ldots \; \bar{\rho}_N]', \quad \alpha = \max_{i=1,\ldots,N} \alpha_i$$

and

$$I(\delta, j) = [\delta \; \delta \; \ldots \; \delta]' \in R^j$$

for any positive integer $j > 0$ and real number δ, the main stability result of the proposed approach can now be stated.

Theorem 1. *Under the assumptions of Lemma* 1 *and Assumption* 3 *if*

(1) $\gamma_i(\bar{\rho}, I(\bar{\rho}_d, N_\mathrm{p}), N_\mathrm{p}) < \bar{\rho}_i(1 - \alpha_i)$,
(2) *there exist* $0 \leqslant \alpha_g < 1$, $\alpha_d \geqslant 0$, $\tilde{\varepsilon} > 0$ *such that* $\forall \rho_i \leqslant \bar{\rho}_i$, $\gamma_i(\rho, I(\varepsilon, j), j) < (\alpha_g \rho_M + \alpha_d \varepsilon)(1 - \alpha)$, $\forall \varepsilon \leqslant \tilde{\varepsilon}, \forall j > 0$, *with*

$$\rho = [\rho_1 \; \rho_2 \; \ldots \; \rho_N]', \quad \rho_M = \max_{i=1,\ldots,N} \rho_i.$$

Then,

(i) *there exist* $\rho_i^0 > 0$ *such that* $\forall x_{i0}^p \in B_{\rho_i^0}$, $x_i^p(nN_\mathrm{p}) \in B_{\tilde{\rho}_i}$, $\forall n \geqslant 0$;
(ii) *the origin is an asymptotically stable fixed point of the perturbed closed-loop system* (1), (9) *with*

$$\tilde{\rho}_i := \beta_i \bar{\rho}_i + \max_{j=0,\ldots,N_\mathrm{p}-1} \gamma_i(\bar{\rho}, I(\bar{\rho}_d, j), j).$$

The result of the theorem is rather conservative, due to the need to consider bounds on the mutual influence between subsystems, their unmodelled dynamics and the effect of the disturbances. However, these bounds could be relaxed when partial information can be exchanged between subsystems. Moreover, in order to reduce the conservativeness inherent to any robust open-loop minimization based *MPC* algorithm, one could resort to min–max closed-loop strategies. For a discussion on this point see e.g. Magni and Scattolini (2005).

4. Example

Consider the following second order system composed of two subsystems S_1 and S_2:

$$S_1 : x_1^p(k+1) = \sqrt{x_1^p(k)^2 + 1} + u_1(k) - 1 + \eta_1 x_2^p(k) + d_1(k), \quad x_1(0) = x_{10},$$

$$S_2 : x_2^p(k+1) = \mathrm{e}^{-\sin(x_2^p(k))} + u_2(k) - 1 + \eta_2 x_1^p(k) + d_2(k), \quad x_2(0) = x_{20},$$

where the "nominal" part of S_1, S_2 is given by the Lipschitz functions

$$f_1(x_1^p, u_1) = \sqrt{x_1^{p2} + 1} + u_1 - 1,$$
$$f_2(x_2^p, u_2) = \mathrm{e}^{-\sin(x_2^p)} + u_2 - 1,$$

while their mutual influence is described by $g_1(x^p) = \eta_1 x_2^p$, $g_2(x^p) = \eta_2 x_1^p$.

As for the disturbances, they are assumed to be the states of the following asymptotically stable first order systems

$$d_i(k+1) = \gamma_i d_i(k), \quad d_i(0) = d_{i0}, \quad i = 1, 2.$$

Finally, the control variables are required to fulfill the following constraints:

$$-0.2 \leqslant u_i(k) \leqslant 0.5, \quad i = 1, 2.$$

The *MPC* algorithm described in Section 3 has been used in a number of simulation experiments with initial conditions $x_{i0} = d_{i0} = 1$, $i = 1, 2$, and with performance indices characterized by $N_\mathrm{p} = 5$, $Q_i = R_i = 1$, $i = 1, 2$. As for the disturbance dynamics, it has been defined by $\gamma_i = 0.9$, $i = 1, 2$. Finally, the contraction constraints have been chosen as $\alpha_i = 0.9$, $i = 1, 2$.

In Fig. 1 the transients of the state and control variables are reported when $\eta_i = \bar{\eta}, i = 1, 2$, with $\bar{\eta} = \{0, 0.2, 0.4\}$.

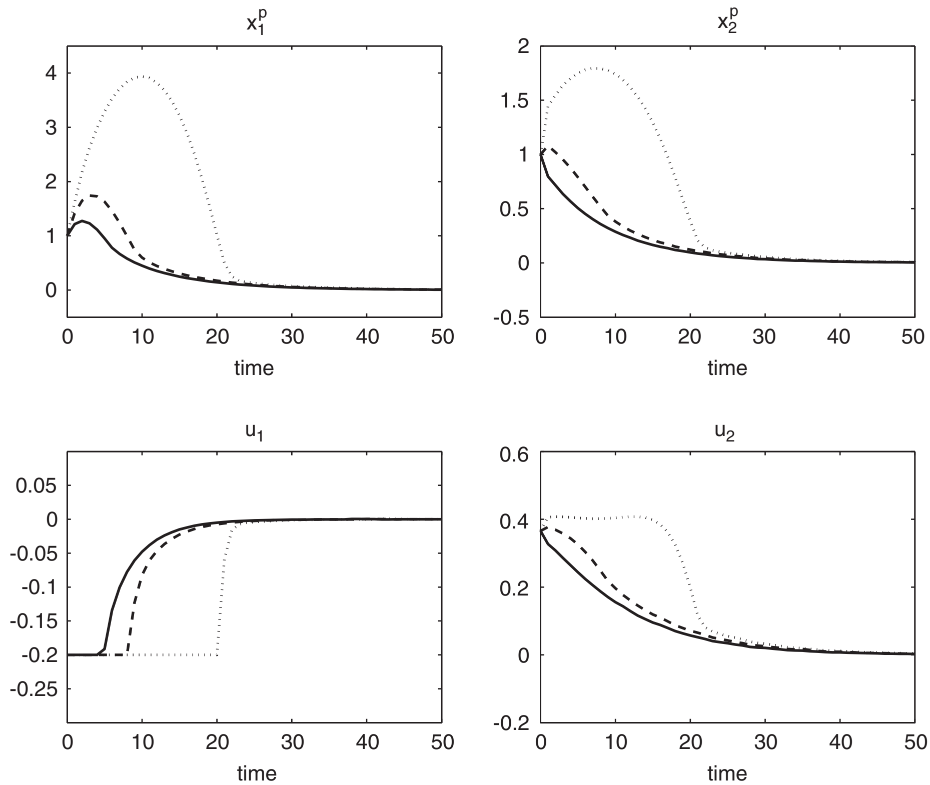

Fig. 1. Transients of the state and control variables for $\bar{\eta}=0$ (continuous line), $\bar{\eta}=0.2$ (dashed line) and $\bar{\eta}=0.4$ (dotted line).

Note that the stability of the origin of the closed-loop system is always achieved, while the performances decrease when the interaction ($\bar{\eta}$) increases. Note also that constraint on the control variable u_1 is active in the initial instants of the transients. Finally with $\bar{\eta}\geqslant 0.5$, starting from the same initial conditions, the state transients diverge.

5. Conclusions

The decentralized predictive control algorithm presented in this note can be extended in several directions. Among them, the output feedback case and its modifications when partial information can be exchanged between local control laws appear to be of interest.

Appendix.

Proof of Lemma 1. In view of Assumptions 1–4 one has

$$\begin{aligned}
&|x_i^p(nN_{\rm p}+k)-\bar{x}_i(nN_{\rm p}+k)|\\
&\quad\leqslant L_{f_i}|x_i^p(nN_{\rm p}+k-1)-\bar{x}_i(nN_{\rm p}+k-1)|\\
&\qquad+\sum_{j=1}^{N}L_{ij}\{|\bar{x}_j(nN_{\rm p}+k-1)|\\
&\qquad+|x_j^p(nN_{\rm p}+k-1)-\bar{x}_j(nN_{\rm p}+k-1)|\}\\
&\qquad+|d_i(nN_{\rm p}+k-1)|\\
&\quad\leqslant L_{f_i}|x_i^p(nN_{\rm p}+k-1)-\bar{x}_i(nN_{\rm p}+k-1)|\\
&\qquad+\sum_{j=1}^{N}L_{ij}\{\beta_j|x_j^p(nN_{\rm p})|\\
&\qquad+|x_j^p(nN_{\rm p}+k-1)-\bar{x}_j(nN_{\rm p}+k-1)|\}\\
&\qquad+|d_i(nN_{\rm p}+k-1)|.
\end{aligned}$$

Finally by iterating backwards the right-hand-side of this expression and by recalling (8) the result follows.

Proof of Theorem 1. In view of Lemma 1, for $i=1,\ldots,N$,

$$\begin{aligned}
&|x_i^p(nN_{\rm p}+N_{\rm p})|-|\bar{x}_i(nN_{\rm p}+N_{\rm p})|\\
&\quad\leqslant|x_i^p(nN_{\rm p}+N_{\rm p})-\bar{x}_i(nN_{\rm p}+N_{\rm p})|\\
&\quad\leqslant\gamma_i(\bar{\rho},I(\bar{\rho}_d,N_{\rm p}),N_{\rm p}).
\end{aligned}\tag{10}$$

Hence, from (6) and condition (1) of the theorem

$$|x_i^p(nN_{\rm p}+N_{\rm p})|\leqslant\alpha_i|x_i^p(nN_{\rm p})|+\gamma_i(\bar{\rho},I(\bar{\rho}_d,N_{\rm p}),N_{\rm p})\tag{11}$$

and

$$\begin{aligned}
|x_i^p(nN_{\rm p})|&\leqslant\alpha_i^n\rho_i^0+\frac{\gamma_i(\bar{\rho},I(\bar{\rho}_d,N_{\rm p}),N_{\rm p})}{1-\alpha_i}\\
&\leqslant\rho_i^0+\frac{\gamma_i(\bar{\rho},I(\bar{\rho}_d,N_{\rm p}),N_{\rm p})}{1-\alpha_i}.
\end{aligned}$$

So, in order to guarantee

$$|x_i^p(nN_{\rm p})|<\bar{\rho}_i,\quad\forall n\geqslant 0,\tag{12}$$

it is sufficient to have

$$\rho_i^0 + \frac{\gamma_i(\bar{\rho}, I(\bar{\rho}_d, N_p), N_p)}{1-\alpha_i} < \bar{\rho}_i,$$

and this is true if and only if

$$\rho_i^0 < \bar{\rho}_i - \frac{\gamma_i(\bar{\rho}, I(\bar{\rho}_d, N_p), N_p)}{1-\alpha_i},$$

but in view of condition (1) this holds true with

$$\rho_i^0 > 0,$$

and then (i) is satisfied.

We now prove that $x_i^p(k) \in B_{\tilde{\rho}_i}, \forall k > 0$. To this end with the same arguments used to derive (10), it is easy to show that, $\forall j \in [0, N_p - 1]$,

$$|x_i^p(nN_p + j)| \leqslant |\bar{x}_i(nN_p + j)| + \gamma_i(\bar{\rho}, I(\bar{\rho}_d, j), j), \tag{13}$$

but from Assumption 4 and recalling that $\bar{x}_i(nN_p) = x_i^p(nN_p)$

$$\begin{aligned}|\bar{x}_i(nN_p + j)| &\leqslant \beta_i|\bar{x}_i(nN_p)| \\ &= \beta_i|x_i^p(nN_p)|, \quad \forall j \in [0, N_p - 1].\end{aligned}$$

Hence

$$|x_i^p(nN_p + j)| \leqslant \beta_i|x_i^p(nN_p)| + \gamma_i(\bar{\rho}, I(\bar{\rho}_d, j), j). \tag{14}$$

From (12) and condition (2), $\forall n \in Z_+$ and $\forall j \in [0, N_p - 1]$ it follows that

$$\begin{aligned}&|x_i^p(nN_p + j)| \\ &\quad \leqslant \beta_i\bar{\rho}_i + \gamma_i(\bar{\rho}, I(\bar{\rho}_d, j), j) \\ &\quad \leqslant \beta_i\bar{\rho}_i + \max_{j=0,\dots,N_p-1} \gamma_i(\bar{\rho}, I(\bar{\rho}_d, j), j) = \tilde{\rho}_i,\end{aligned}$$

so that (i) of Definition 1 is satisfied.

In order to prove (ii) of Definition 1 first we note that if $d_i(k) \to 0$ as $k \to \infty$, then for any $\varepsilon > 0$ there exists a finite $\bar{n}_\varepsilon \in Z_+$ so that $|d_i(k)| \leqslant \varepsilon$, $\forall k \geqslant \bar{n}_\varepsilon N_p$. In view of (11) one has

$$|x_i^p(\bar{n}_\varepsilon N_p + N_p)| \leqslant \alpha_i|x_i^p(\bar{n}_\varepsilon N_p)| + \gamma_i(\bar{\rho}, I(\varepsilon, N_p), N_p)$$

and, $\forall l \in Z_+$, letting

$$\bar{\rho}_i^{\max}(\bar{n}_\varepsilon N_p, l) := \max_{k=0,\dots,l-1} |x_i^p((\bar{n}_\varepsilon + k)N_p)|,$$

$$\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, l) := \max_{i=1,\dots,N} \bar{\rho}_i^{\max}(\bar{n}_\varepsilon N_p, l),$$

then

$$\begin{aligned}&|x_i^p((\bar{n}_\varepsilon + h)N_p)| \\ &\quad < \alpha_i^h|x_i^p(\bar{n}_\varepsilon N_p)| \\ &\qquad + \frac{\gamma_i(I(\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, h), N), I(\varepsilon, N_p), N_p)}{1-\alpha_i}.\end{aligned} \tag{15}$$

Then, in view of (15) and condition (2) there exists $\tilde{\varepsilon}$ such that $\forall \varepsilon < \tilde{\varepsilon}$

$$\begin{aligned}&\bar{\rho}^{\max}((\bar{n}_\varepsilon + h)N_p, 0) \\ &\quad < \alpha^h\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, 0) + \alpha_g\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, h) + \alpha_d\varepsilon \\ &\quad \leqslant (\alpha^h + \alpha_g)\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, h) + \alpha_d\varepsilon.\end{aligned}$$

In view of condition (2) there exists a positive integer l such that $\bar{\alpha}_g(h) := \alpha^h + \alpha_g < 1$, $\forall h \geqslant l$,

$$\begin{aligned}&\bar{\rho}^{\max}((\bar{n}_\varepsilon + l)N_p, l) \\ &\quad = \max_{\varphi=0,\dots,l-1}\{\bar{\rho}^{\max}((\bar{n}_\varepsilon + l + \varphi)N_p, 0)\} \\ &\quad \leqslant \max_{\varphi=0,\dots,l-1} \bar{\alpha}_g(l)\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, l + \varphi) + \alpha_d\varepsilon.\end{aligned}$$

Moreover,

$$\begin{aligned}&\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, l + \varphi) \\ &\quad = \max\{\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, l + \varphi - 1), \\ &\qquad \bar{\rho}^{\max}((\bar{n}_\varepsilon + l + \varphi - 1)N_p, 0)\} \\ &\quad \leqslant \max\{\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, l + \varphi - 1), \\ &\qquad \bar{\alpha}_g(l + \varphi - 1)\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, l + \varphi - 1) + \alpha_d\varepsilon\}\end{aligned}$$

so that

$$\begin{aligned}&\lim_{\varepsilon\to 0} \bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, l + \varphi) \\ &\quad \leqslant \lim_{\varepsilon\to 0} \bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, l + \varphi - 1) \leqslant \cdots \leqslant \lim_{\varepsilon\to 0} \bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, l),\end{aligned}$$

and then

$$\lim_{\varepsilon\to 0} \bar{\rho}^{\max}((\bar{n}_\varepsilon + l)N_p, l) \leqslant \lim_{\varepsilon\to 0} \bar{\alpha}_g(l)\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, l)$$

and

$$\lim_{\varepsilon\to 0} \bar{\rho}^{\max}((\bar{n}_\varepsilon + ml)N_p, l) \leqslant \lim_{\varepsilon\to 0} \bar{\alpha}_g^m(l)\bar{\rho}^{\max}(\bar{n}_\varepsilon N_p, l),$$

so that

$$\lim_{\varepsilon\to 0} \lim_{m\to\infty} \bar{\rho}^{\max}((\bar{n}_\varepsilon + ml)N_p, l) = 0.$$

Hence

$$\lim_{n\to\infty} \lim_{d_i\to 0} |x_i^p(nN_p)| = 0, \quad \forall i = 1, \dots, N. \tag{16}$$

Finally, in view of Lemma 1, $j \geqslant 0$, $i = 1, \dots, N$,

$$\begin{aligned}&|x_i^p(nN_p + j)| \\ &\quad \leqslant |x_i^p(nN_p + j) - \bar{x}_i(nN_p + j)| + |\bar{x}_i(nN_p + j)| \\ &\quad \leqslant \gamma_i(X^p(nN_p), D(nN_p, j), j) + \beta_j|x_i^p(nN_p)|\end{aligned}$$

and from (16) and Assumption 4

$$\begin{aligned}&\lim_{n\to\infty} \lim_{d_i\to 0} |x_i^p(nN_p + j)| = 0, \quad \forall j \in [0, N_p - 1], \\ &\quad \forall i = 1, \dots, N,\end{aligned}$$

so that (ii) of Definition 1 is satisfied and the origin is an asymptotically stable fixed point of the perturbed closed loop system (1)–(9).

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