

Dear students,

in order to make the solution of exercise 2 univocal, I recommend to follow, besides the standard rules, the following ones:

- **When writing down the variables in the tableau use the following order:** first the x variables (x_1, x_2, \dots in case you replaced $x_1 = x_{1+} - x_{1-}$ write first x_{1-} and then x_{1+}), then the delta variables, after the slack variables and finally, for phase 1, the y variables.

- **When writing down the equality constraints involving slack variables:** in case the constant is 0, please chose the formulation with "+slack" instead of "-slack".

Example: $x_{1+} - x_2 - s_1 = 0$, use $-x_{1-} - x_2 + s_1 = 0$ instead.

- **In case there are more identical indicator vectors** (with the 1 on the same row) select the one further to the left.

- **In case there are more indicator vectors than number of constraints** (number of rows besides the first one of the cost) please choose the non identical ones further to the left. Example: we have 3 constraints and 6 optimization variables and 4 columns are indicator vectors. Then choose the first (from the left) 3 non-identical indicator vectors and set the corresponding variables as basic variables (write them on the left of the corresponding rows).

- **At the start of phase 1** if there are less indicator vectors than number of constraints, subtract all the rows to the first one to make the columns of the variables y indicator vectors. After this procedure select as basic variables according to the previous rule the indicator columns to the further left and then proceed with the bland's rule as always.

- **At the start of phase 2** it may happen not to have a basic variable for each row of the constraints (for example when one of the slack variables s appears with a minus). In that case proceed as follows: take the first row which does not have an associated basic variable and take the first column of this row which has a positive number. Perform the pivoting with respect to this number. Continue this procedure until you have all rows with an associated basic variable. After that follow the Bland's rules. *Note that this rule overrides the one in the slides where it was suggested to bring at the beginning of phase 2 in the basis exactly the BVs obtained at the end of phase 1.*

- **Choosing the pivoting:** according to the Bland's rule the pivoting is done by taking the element which corresponds to the smallest ratio b/a (with $a \geq 0$). In case 2 rows lead to the same b/a please always chose the first one.

- **Branch and bound** please order the variables according to their number δ_1, δ_2 , etc. Then, when you depict the tree, branch first with respect to δ_1 , then for δ_2 , etc. *Moreover, depict $\delta = 0$ on the left and $\delta = 1$ on the right of a branch.* Finally, in solving the branch and bound, please use the depth first approach meaning you first

explore a branch till the end (or until you don't need to explore it any further down) and only after move to the next branch.

Best regards,
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