Advanced Automation and Control

September 2, 2020

Surname ____________________ Name ____________________

Part I - Optimization (Prof. D.M. Raimondo)

1. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{array}{ll} \max_{x_1,\delta_1,\delta_2} & -x_1 + 2\delta_2 \\ & -x_1 \leq -\delta_1 + 0.5 \\ & -\delta_1 + \delta_2 \leq 0 \\ & \delta_1, \delta_2 \in \{0, 1\} \\ & x_1 \leq 0 \end{array}$$

2. Mr. Sedinho, coach of New Team football club is preparing the next match. While 10/11 players are decided, he is still thinking about the tactical module ($4-4-2$ or $3-5-2$) and which player to utilize. In case of $4-4-2$ he will need a defender, while, in case of $3-5-2$ a midfielder. The table below lists the players he can choose. One can play both as midfielder and as defender but with different added values to the team. Unfortunately, these players cannot play the entire match and will have to be replaced. If a player is replaced, another in the table (compatible with the module) will play for a number of minutes equal to 90 minus the minutes played by the first player. Assume that Mr. Sedinho does not change module over the game. Moreover, consider that Mellow is not willing to be a substitute player.

   Please help Mr. Sedinho in choosing the tactic module, the titular player and the substitute player in order to maximize the added value of the team. Note that, the added value of a player gets multiplied by the number of minutes played. Please formulate the problem above as an optimization program.

   Extra question - 1.5 points: What would be in your opinion the best choice to maximize the added value of the team? What would be the optimal cost? Please answer in the sheet to be delivered at the beginning of point 2 of Exercise 2 motivating the answer.

Player	Time he can play	Role he can play	Value as midfielder	Value as defender
Ross	70 min	D	-	1
Mellow	65 min	M/D	0.5	0.9
Becker	55 min	M	1.1	-

3. Consider the following optimisation problem

$$\begin{array}{ll} \min_x & f(x) \\ & \sin(x) \leq 1 \\ & x^2 \leq 16 \\ & \cos(x) \geq -1 \end{array}$$

   where $f(x) = \begin{cases} -3x - 3, & \text{if } x \leq -1 \\ x + 1, & \text{if } -1 \leq x \leq 0 \\ -x + 1, & \text{if } 0 \leq x \leq 1 \\ 3x - 3, & \text{if } 1 \leq x \end{cases}$

   3.1 Depict the cost function and indicate if it is convex (motivate the answer).

   3.2 Depict the feasibility domain of the problem. Is it convex (motivate the answer)?

   3.3 Indicate if the optimisation problem is convex (motivate the answer).

4. For the directed network in the figure below

   - compute all shortest paths from vertex 1 to all other vertices.

- Assume you have the chance to add an edge of cost 1 between two nodes. Where would you add it to make the node that was not reachable from vertex 1, now reachable with a minimum cost?
- After having added such edge, can you remove another one while keeping all nodes reachable from node 1? If possible, what is the edge with the highest cost that can be removed? Please discuss.