Advanced Automation and Control

June 22, 2021

Surname _____ Name _____

Part I - Optimization (Prof. D.M. Raimondo)

1. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{aligned} \max_{x_1,\delta_1,\delta_2} \quad & 10x_1 + \delta_1 + \delta_2 \\ & \delta_1 \geq x_1 \\ & x_1 + \delta_1 + \delta_2 = 1 \\ & x_1 \geq 0 \\ & \delta_1, \delta_2 \in \{0, 1\} \end{aligned}$$

2. Professor Haiwelt has to make a trip using an electric car that will take him from Milan to Zurich. The vehicle's battery is constrained to operate between 0% and 100% of State of Charge (SOC). In order to reach the destination, it is possible to choose between route A and B (see the details in the table below). In particular, Route A is slower but has the advantage of being shorter and does not need to recharge the car. However it has a fixed cost of 25€ (highway toll). For route A, the average battery consumption is 0.25% of SOC every km. Route B is faster and does not require the payment of a highway toll. The average SOC consumption can vary from 0.35%/km to 0.4%/km depending on whether the cruise control is respectively set in eco or sport mode. After 250km of route B, there is a charging station where it is possible to recharge the car at a speed of 1% SOC per minute at the price of 0.3 €/min.

Route	**Length (km)**	**Velocity(km/h)**	**Consumption (SOC%/km)**	**Toll (€)**
A	350	70	0.25	25
B	450	100 (eco) or 120 (sport)	0.35 (eco) or 0.4 (sport)	-

Please help the professor get to Zurich by minimizing travel cost. Also consider that for safety reasons the car must arrive at the charging station and at the final destination with at least 10% SOC. Assume also that the car starts its journey with $SOC = 100\%$ and that the trip must be completed within 5 hours and 40 minutes.

3. Consider the following optimisation problem

$$\begin{aligned} \min_x \quad & f(x) \\ & (x-4)^2 \geq 100 \end{aligned}$$

where $f(x) = \begin{cases} (x+2)^2, & \text{if } x \leq -1 \\ x+2, & \text{if } -1 \leq x \leq 1 \\ 3x^2, & \text{if } 1 \leq x \end{cases}$

3.1 Depict the cost function and indicate if it is convex (motivate the answer also by checking convexity analitically).

3.2 Depict the feasibility domain of the problem. Is it convex (motivate the answer)?

3.3 Indicate if the optimisation problem is convex (motivate the answer).

4. Consider the following Linear Program

$$\begin{aligned} \max \quad & x_1^2 + x_1x_2 + 9x_2^2 - 1.5x_1x_2 + 8x_1 - 6 \\ & x_1 \geq x_2 \\ & x_1 + 3x_2 = -4 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{aligned}$$

Compute the corresponding dual program.