Advanced Automation and Control

February 23, 2023

Surname ____________________ Name ____________________

Optimization (Prof. D.M. Raimondo)

- Ex. 1 (**8 points)** Consider the the following MILP problem.

$$
\begin{aligned}
\min_{x_1,\delta_1,\delta_2} \quad & \delta_2 + 3x_1 \\
& 0.5\delta_1 + \delta_2 \geq 1 \\
& x_1 \geq \delta_2 \\
& \delta_1, \delta_2 \in \{0,1\} \\
& x_1 \geq 0
\end{aligned}
$$

  The relaxation of node 0 does not provide a feasible solution for the original problem but only a lower bound equal to +2. With this information available, please perform the remaining steps necessary to find the optimal solution using the branch and bound algorithm.

- Ex. 2 (**10 points)** The famous soccer player Z.I. will be one of the guests of the next Sanremo Music Festival. The festival, held in Sanremo, Italy, lasts 3 evenings (Tuesday to Thursday), 5 hours each evening. Z.I. he will be present for the three nights. He has yet to decide how many hours he will be present per day, but certainly at least 1 hour/day. The cachet of Z.I. is $25K€$ per hour. On Thursday Z.I. has an important soccer match in the afternoon in Milan for which he has to properly train on Tuesday and Wednesday. Note that, he will attend the festival also on Thursday (after the match). For the training Z.I. can:

  – practice in Milan (where he lives) and travel to Sanremo to attend the festival;

  – practice directly in Sanremo.

  For Tuesday and Wednesday nights, in case Z.I. he decides to train in Sanremo, he will sleep in Sanremo (on Thursday, after the festival, he will definitely return to Milan). The cost of the suite is $4K€$/night. Note that Z.I. will not train in different cities on different days - only one choice is possible (either always Milan or always Sanremo). To travel between Milan and Sanremo, he has 2 choices:

  – by car, takes 3hrs, costs $1K$ €;

  – by helicopter, takes 1hr, costs $2.2K$ €.

  The costs are for a one way trip. The travel choice will be the same for all the trips. Note that, he will not pay for the following expenses (the festival organization will take care of that):

  – travel costs from Milan to Sanremo for the first day of the festival (he gets there on Tue);

  – (eventual) travel costs to the stadium from Sanremo the day of the game;

  – travel costs to Milan from Sanremo after the festival ends.

  He will have to pay for the other trips. Z.I. has allocated 12 hours a day for Tuesday and Wednesday for the sum of travel, training and festival time, but can train a maximum of 8 hours a day. Note that even though some trips are paid for by the festival organization, they still count against this time constraint. Keep in mind that, for Thursday, Z.I. does not have this time limit. Finally, for Thursday's game, Z.I. will play only if he has trained for at least 14 hours in total between Tuesday and Wednesday. In that case he will receive a bonus of $60K€$.

  Please help Z.I. in planning his week with the aim of maximizing his profits.

- Ex. 3 (**6 points**) Consider the following optimisation problem

$$\begin{array}{ll} \max_x & f(x) \\ & x^2 \geq 0 \\ & x \leq -1 \\ & x \geq 1 \end{array}$$

where $f(x) = \begin{cases} x + \pi/2 & \text{if } x < -\pi/2 \\ \cos(x), & \text{if } -\pi/2 \leq x \leq \pi/2 \\ -x + \pi/2, & \text{if } x > \pi/2 \end{cases}$

3.1 Depict the cost function and indicate whether it is convex, concave, or neither convex nor concave (motivate the answer).

3.2 Depict the feasibility domain of the problem. Is it convex (motivate the answer)?

3.3 Indicate if the optimisation problem is convex (motivate the answer).

Ex. 4 (**7 points**) Consider the mobile robot problem in the figure below. The square in green indicates the goal while the black squares the obstacles. Assume the robot can move {up,down,left,right} and that each action has cost 1. Assume also that an extra action {stay} is available at the goal only with cost 0.5.

4.1 What is the minimum horizon to guarantee the attainment of the goal from any initial condition?

4.2 For $N = 2$ please formulate the dynamic program (write down the stage cost and the terminal cost) and solve the first step (J_{N-1} and μ_{N-1}) only.

4.3 Please provide the cost and the optimal input sequence needed to attain the goal starting from the position of the robot (blue square) given in the figure.