Advanced Automation and Control

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Surname _____ Name _____

Optimization (Prof. D.M. Raimondo)

- Ex. 1 (**8 points)** Please solve the following MILP problem using the branch and bound algorithm

$$\begin{aligned} \min_{x_1,\delta_1,\delta_2} \quad & \delta_2 + 3x_1 \\ & 0.5\delta_1 + \delta_2 \geq 1 \\ & x_1 \geq \delta_2 \\ & \delta_1, \delta_2 \in \{0,1\} \\ & x_1 \geq 0 \end{aligned}$$

- Ex. 2 (**10 points)** Mr. Grecchi owes a company that currently counts 90 employees (salary 80K€/employee). Each employee makes on average 100 units/year which are sold at 1000€ each. The company is considering a possible reorganization. One possibility consists in training some employees (at most 20) so to improve their skills. The training would last one year and would cost to the company 10K€/employee. During this time, the employees will make no products. On the other side, after the training, the productivity of these workers would raise to 130 units/year. As a motivation, the trained employees will receive a 10% salary increase starting already from the 1st year. The company has also the possibility of purchasing robots (up to 10) for a fixed cost of 1.3M€/robot. Their productivity is of 200 units/year. Due to space limitations, if the overall production exceeds the 10700 units/year, it will be necessary to expand the company site (fixed cost 1M€). Finally, in order to reduce the total payroll, Mr. Grecchi has also the option to dismiss some of the employees (up to 20). However, in case more than 10 workers are dismissed, the company could expect some legal disputes whose costs are estimated in around 100K€ (legal class action). Please formulate the optimization problem in order to maximize the profit of the company over the next 10 years assuming no other costs are present and all units produced are sold.

- Ex. 3 (**6 points)** Consider the following optimisation problem

$$\begin{aligned} \min_x \quad & f(x) \\ & x^2 \leq 4 \\ & x^2 \geq 1 \\ & x^3 \geq 0 \end{aligned}$$

where $f(x) = \begin{cases} -2x & \text{if } x < 0 \\ -x, & \text{if } 0 \leq x \leq 2 \\ x^2 - 6, & \text{if } x > 2 \end{cases}$

3.1 Depict the cost function and indicate if it is convex (motivate the answer).

3.2 Depict the feasibility domain of the problem. Is it convex (motivate the answer)?

3.3 Indicate if the optimisation problem is convex (motivate the answer).

Ex. 4 (**7 points)** Consider the following Linear Program

$$\begin{aligned} \min \quad & x_1 - x_2 + x_3 - x_4 \\ & x_1 \geq x_2 \\ & x_2 \leq x_4 \\ & x_3 + x_4 = 1 \\ & x_1 \leq 0 \\ & x_2 \geq 0 \\ & x_3 \leq 0 \\ & x_4 \geq 0 \end{aligned}$$

Compute the corresponding dual program.