

# Advanced Automation and Control

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Surname ----- Name -----

## Part I - Optimization (Prof. D.M. Raimondo)

1. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{aligned} \max_{x_1, \delta_1, \delta_2} \quad & \delta_1 + \delta_2 - x_1 \\ & \delta_1 \geq 0.5 - x_1 \\ & \delta_1 + \delta_2 \leq 0.9 \\ & x_1 \leq 0 \\ & \delta_1, \delta_2 \in \{0, 1\} \end{aligned}$$

2. Mr. Moscatelli is the owner of a company where most of his employees work as door to door sales representatives. Mr. Moscatelli needs to renew the company's car fleet (he needs 15 cars for 8 years). He identified two options (the choice is the same for all the cars):

- Long term car rental. The contract lasts for 4 years. The price per car is 250€/month and includes maintenance and insurance. However, in case of more than 120000Km/year per car, there is an extra cost of 0.05€/Km per car.
- Buy the automobiles. The price is 20000€ plus the costs of insurance and maintenance. These latter amount to 300€/year for the first 4 years of life of the cars. Then, due to the cars aging, the live costs increase to 450€/year.

In order to simplify the problem, actions can be taken only at the beginning of year 1 and at the beginning of year 5. In particular, the company can opt for:

- car rental for 4 years and then car rental again.
- car rental for the first 4 years and cars purchase afterwards or viceversa.
- cars purchase only.

Note that, the purchased cars age according to the following formula  $20000 - 1000\sqrt{20Y}$ , where  $Y$  stands for the numbers of years the cars have been used for. At the end of year 4, or at the end of year 8, the company can sell the cars at this estimated price (the cars are not needed after eight years). Note that the company can, after 4 years, sell the current cars and buy new ones thus avoiding, if convenient, the increase of maintenance costs due to aging. Finally, keep in mind that, due to an estimated increase of the business, the cars utilization (does not matter if rented or purchased) over the years is as follows: 60000 + 20000t Kilometers,  $t = 1, \dots, 4$  and 70000 + 30000(t - 4) Kilometers,  $t = 5, \dots, 8$ .

Please formulate the optimization problem in order to help Mr. Moscatelli in minimizing the company's expenses over the considered 8 years.

3. Consider the following optimisation problem

$$\min_x \quad \begin{aligned} & f(x) \\ & \sin(x) \leq 1 \end{aligned}$$

$$\text{where } f(x) = \begin{cases} -x + \pi, & \text{if } x < \pi \\ \sin(x), & \text{if } \pi \leq x \leq 2\pi \\ 0.8x - 1.6\pi, & \text{if } 2\pi < x \end{cases}$$

**3.1** Depict the cost function and indicate if it is convex (motivate the answer also by checking convexity analitically).

**3.2** Depict the feasibility domain of the problem. Is it convex (motivate the answer)?

**3.3** Indicate if the optimisation problem is convex (motivate the answer).

4. Consider the following Linear Program

$$\begin{aligned} \min \quad & x_1 - x_2 + x_3 \\ & 2x_1 \geq x_2 \\ & x_2 \leq x_3 \\ & x_1 + 3x_2 = -6 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \\ & x_3 \geq 0 \end{aligned}$$

Compute the corresponding dual program.