|Ex|

$$\max \quad x_1 + 3\delta_1 + 2\delta_2$$

$$x_1 + \delta_1 + \delta_2 = 5$$

$$x_1 \geq 0$$

$$\delta_1, \delta_2 \in \{0, 1\}$$

LP relaxation at node 0

$$\max \quad x_1 + 3\delta_1 + 2\delta_2$$

$$x_1 + \delta_1 + \delta_2 = 5$$

$$\delta_1 + s_1 = 1$$

$$\delta_2 + s_2 = 1$$

$$x_1, s_1, s_2, \delta_1, \delta_2 \geq 0$$

How do we solve it?

Phase 1

$$\min \quad y_1 + y_2 + y_3$$

$$x_1 + \delta_1 + \delta_2 + y_1 = 5$$

$$\delta_1 + s_1 + y_2 = 1$$

$$\delta_2 + s_2 + y_3 = 1$$

$$x_1, s_1, s_2, \delta_1, \delta_2, y_1, y_2, y_3 \geq 0$$

	x_1	s_1	s_2	δ_1	δ_2	y_1	y_2	y_3
0	0	0	0	0	0	1	1	1
5	1	0	0	1	1	1	0	0
1	0	1	0	1	0	0	1	0
1	0	0	1	0	1	0	0	1

← subtract to first row all other rows ↓

	x_1	s_1	s_2	δ_1	δ_2	y_1	y_2	y_3
-7	-1	-1	-1	-2	-2	0	0	0
5	1	0	0	1	1	1	0	0
1	0	1	0	1	0	0	1	0
1	0	0	1	0	1	0	0	1

1st column with negative z_F^T

5/1 → pivoting on this element

1/0

1/0

-2	0	-1	-1	-1	-1	1	0	0
5	1	0	0	1	1	1	0	0
1	0	1	0	1	0	0	1	0
1	0	0	1	0	1	0	0	1

↓ pivoting

-1	0	0	-1	0	-1	1	1	0
5	1	0	0	1	1	1	0	0
1	0	1	0	1	0	0	1	0
1	0	0	1	0	1	0	0	1

↓ Pivoting

$\bar{b}$									
	0	0	0	0	0	0	1	1	1
δ_1	5	1	0	0	1	1	1	0	0
s_1	1	0	1	0	1	0	0	1	0
s_2	1	0	0	1	0	1	0	0	1

$\bar{A}$

$\boxed{\text{OK!}}$

I take $\bar{b}$ and $\bar{A}$ to start phase 2 original cost. Remember: $\boxed{MAX}$

$$
\begin{array}{c|c|ccccc}
 & 0 & 1 & 0 & 0 & 3 & 2 \\
\hline
 & 5 & 1 & 0 & 0 & 1 & 1 \\
s_1 & 1 & 0 & 1 & 0 & 1 & 0 \\
s_2 & 1 & 0 & 0 & 1 & 0 & 1 \\
b & & & & & &
\end{array}
\xrightarrow{\text{pivoting}}
\begin{array}{c|c|ccccc}
 & -5 & 0 & 0 & 0 & 2 & 1 \\
\hline
x_1 & 5 & 1 & 0 & 0 & 1 & 1 \\
s_1 & 1 & 0 & 1 & 0 & 1 & 0 \\
s_2 & 1 & 0 & 0 & 1 & 0 & 1
\end{array}
$$

pivoting

$5/1$

$1/1$

$1/0$

first colum with $c_f > 0$

$$
\begin{array}{c|c|ccccc}
 & -7 & 0 & -2 & 0 & 0 & 1 \\
\hline
x_1 & 4 & 1 & -1 & 0 & 0 & 1 \\
\delta_1 & 1 & 0 & 1 & 0 & 1 & 0 \\
s_2 & 1 & 0 & 0 & 1 & 0 & 1
\end{array}
$$

$4/1$

$1/1$

$$\downarrow$$

$$
\begin{array}{c|c|ccccc}
 & -8 & 0 & -2 & -1 & 0 & 0 \\
\hline
x_1 & 3 & 1 & -1 & -1 & 0 & 0 \\
\delta_1 & 1 & 0 & 1 & 0 & 1 & 0 \\
s_2 & 1 & 0 & 0 & 1 & 0 & 1
\end{array}
$$

OK!

Cost = 8

$$x = \begin{pmatrix} x_1 \\ s_1 \\ s_2 \\ \delta_1 \\ \delta_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

Great! The solution of the relaxed LP has $\delta_1 = 1$ and $\delta_2 = 1$ $\longrightarrow$ is the optimal solution of the original MILP!

@ HOME Replace 5 with 5.5 in the constraint $\boxed{x_1 + \delta_1 + \delta_2 = 5.5}$