

Ford-Fulkerson algorithm: an example

Prof. Giancarlo Ferrari Trecate

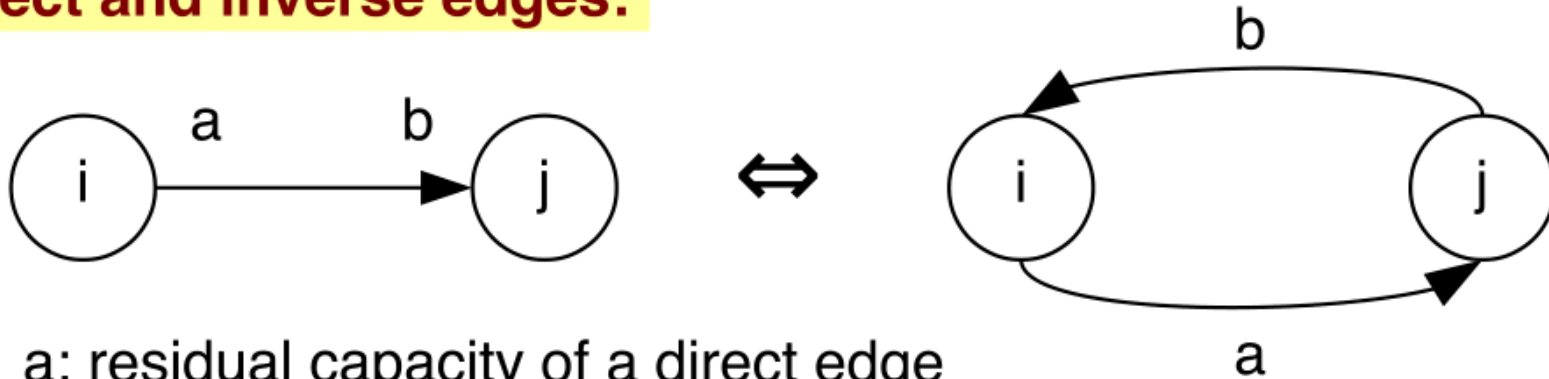
Dipartimento di Ingegneria Industriale e dell'Informazione

Università degli Studi di Pavia

`giancarlo.ferrari@unipv.it`

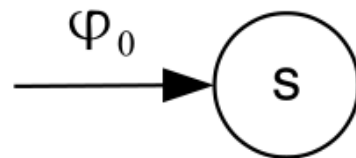
Notation for the residual network

Direct and inverse edges:



a : residual capacity of a direct edge
 b : residual capacity of an inverse edge

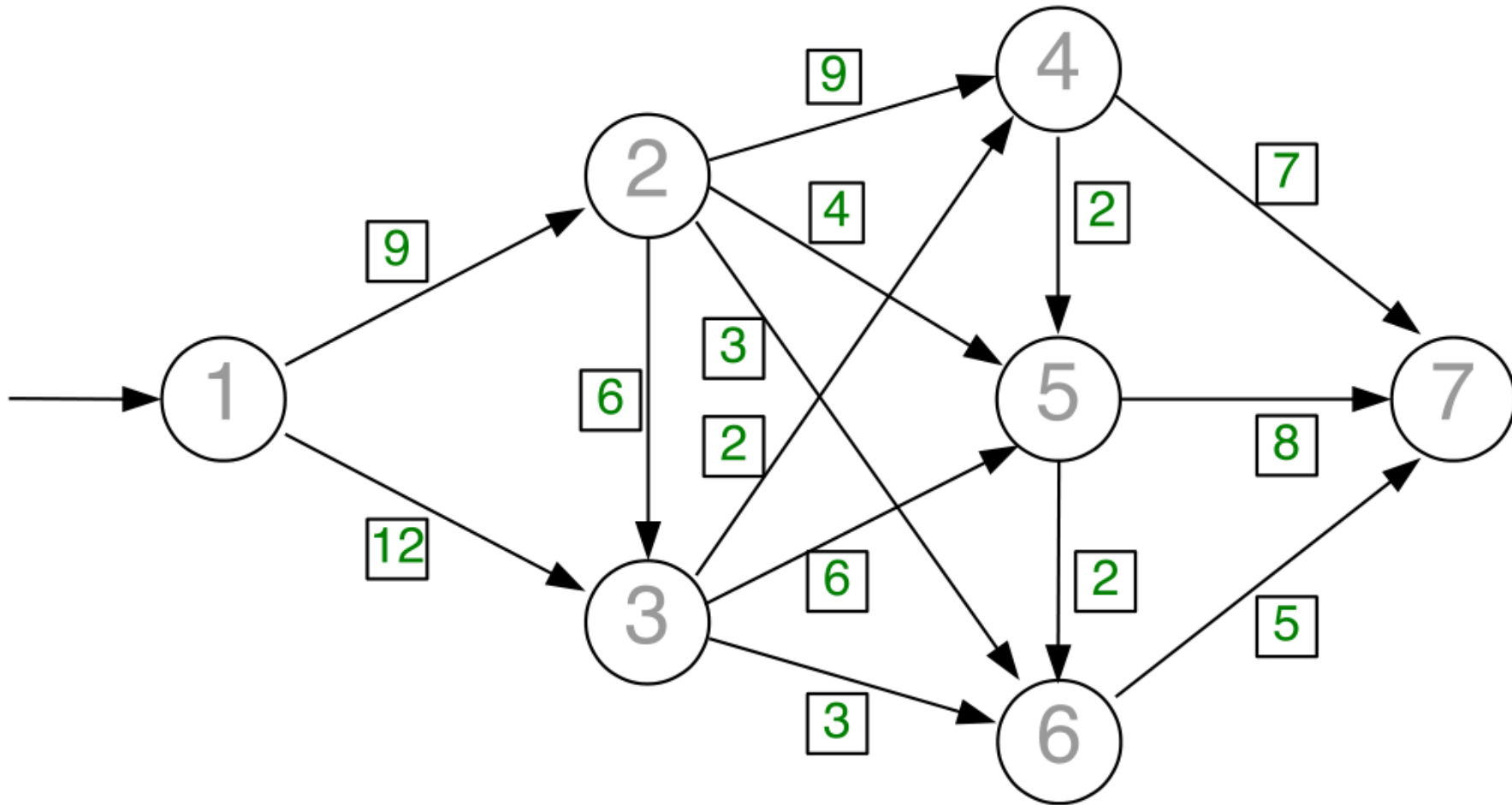
Source node:



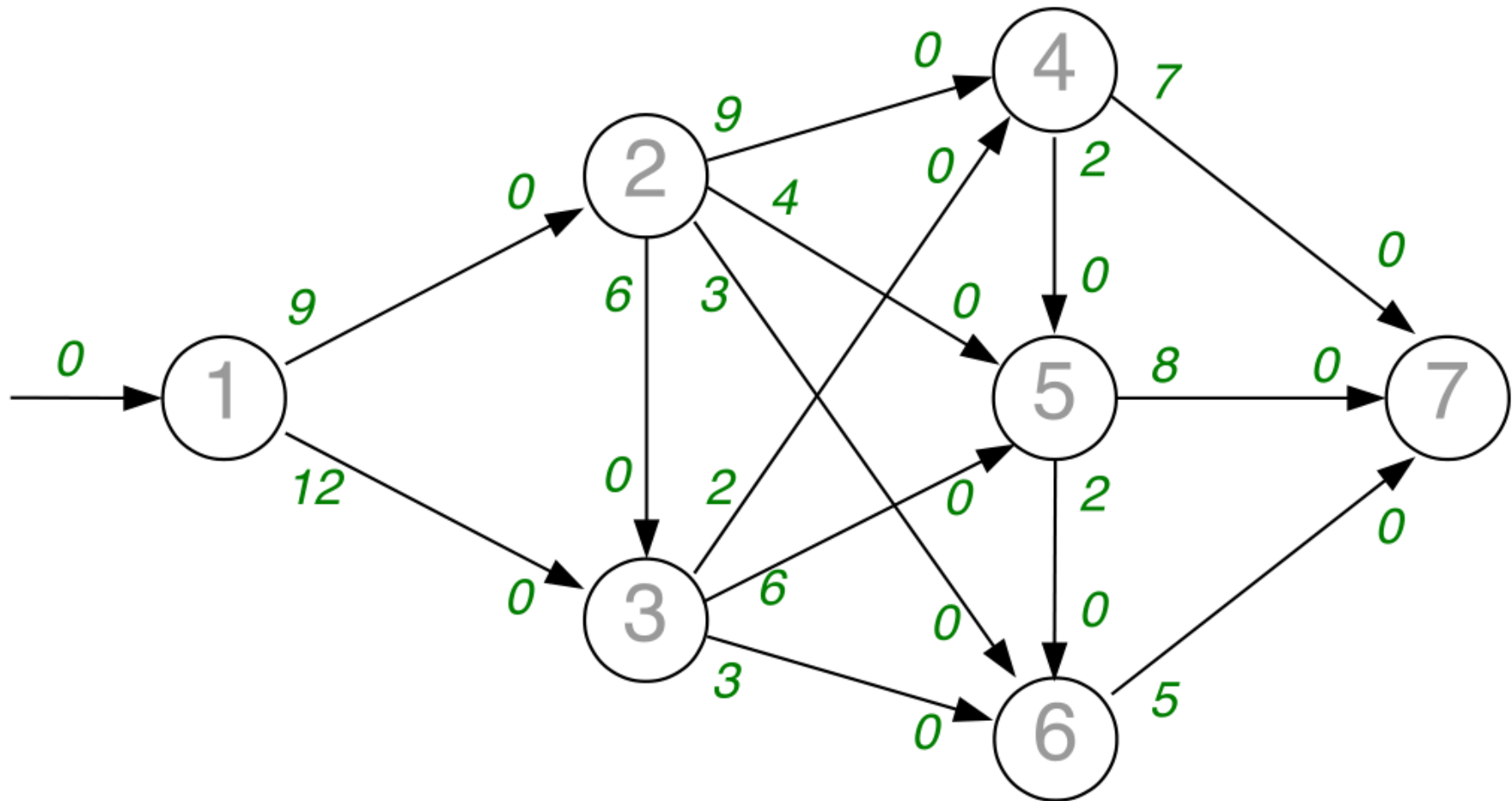
φ_0 : value of the flow

Problem

Maximize the flow from $s=1$ to $t=7$ in the flow network below

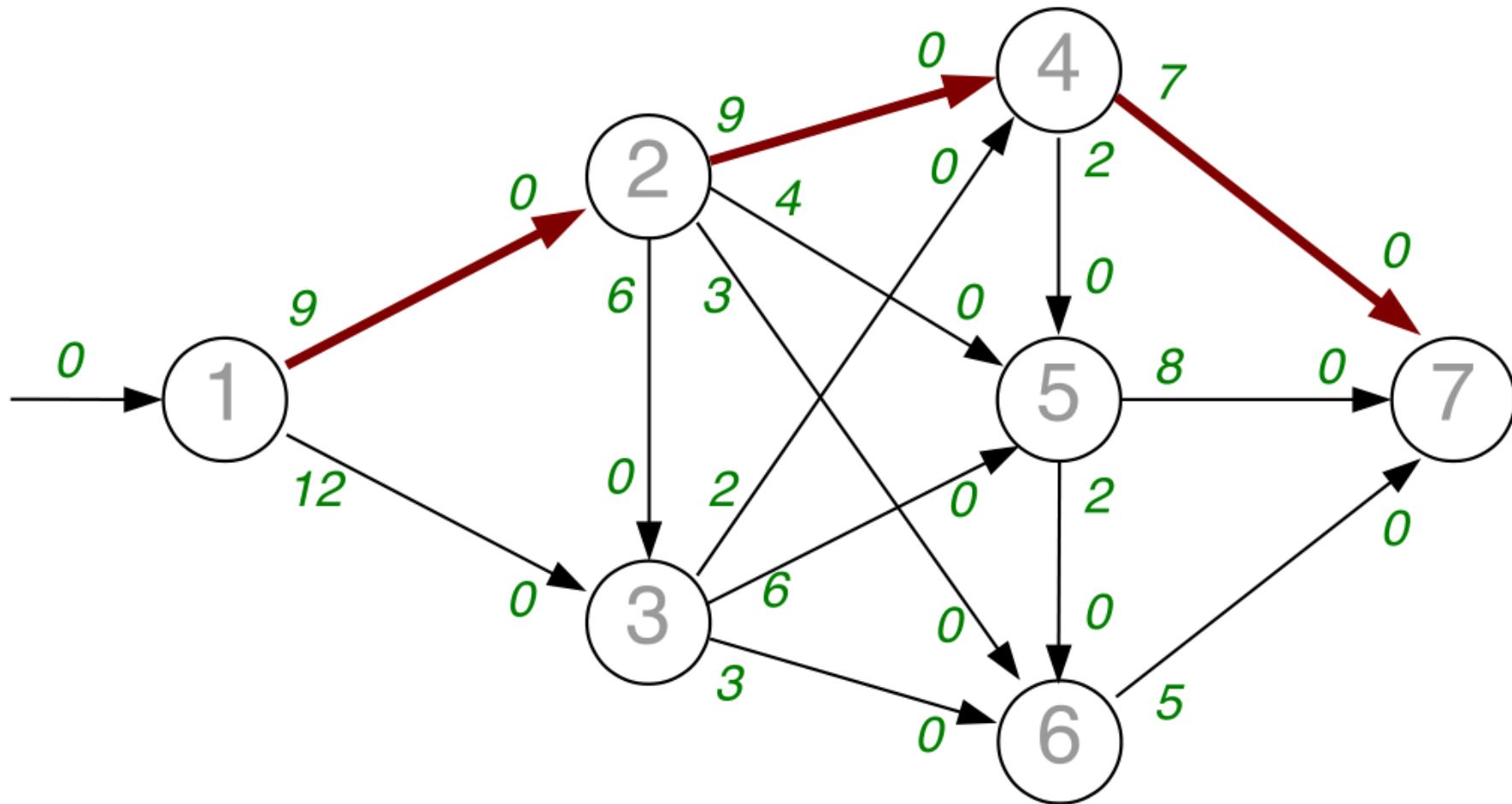


Initialization



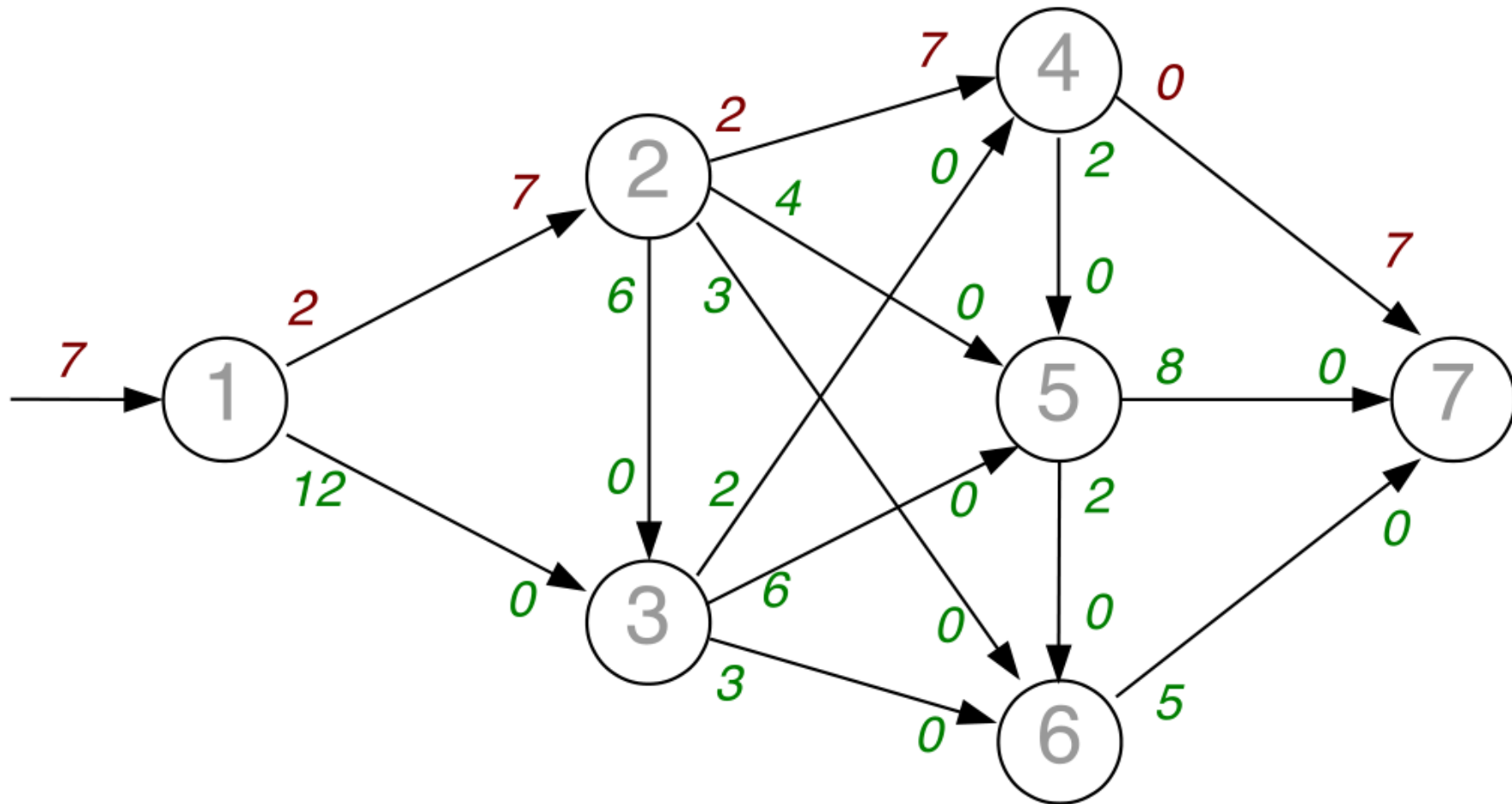
Capacities of inverse edges are equal to the capacities in the original network

Iteration 1



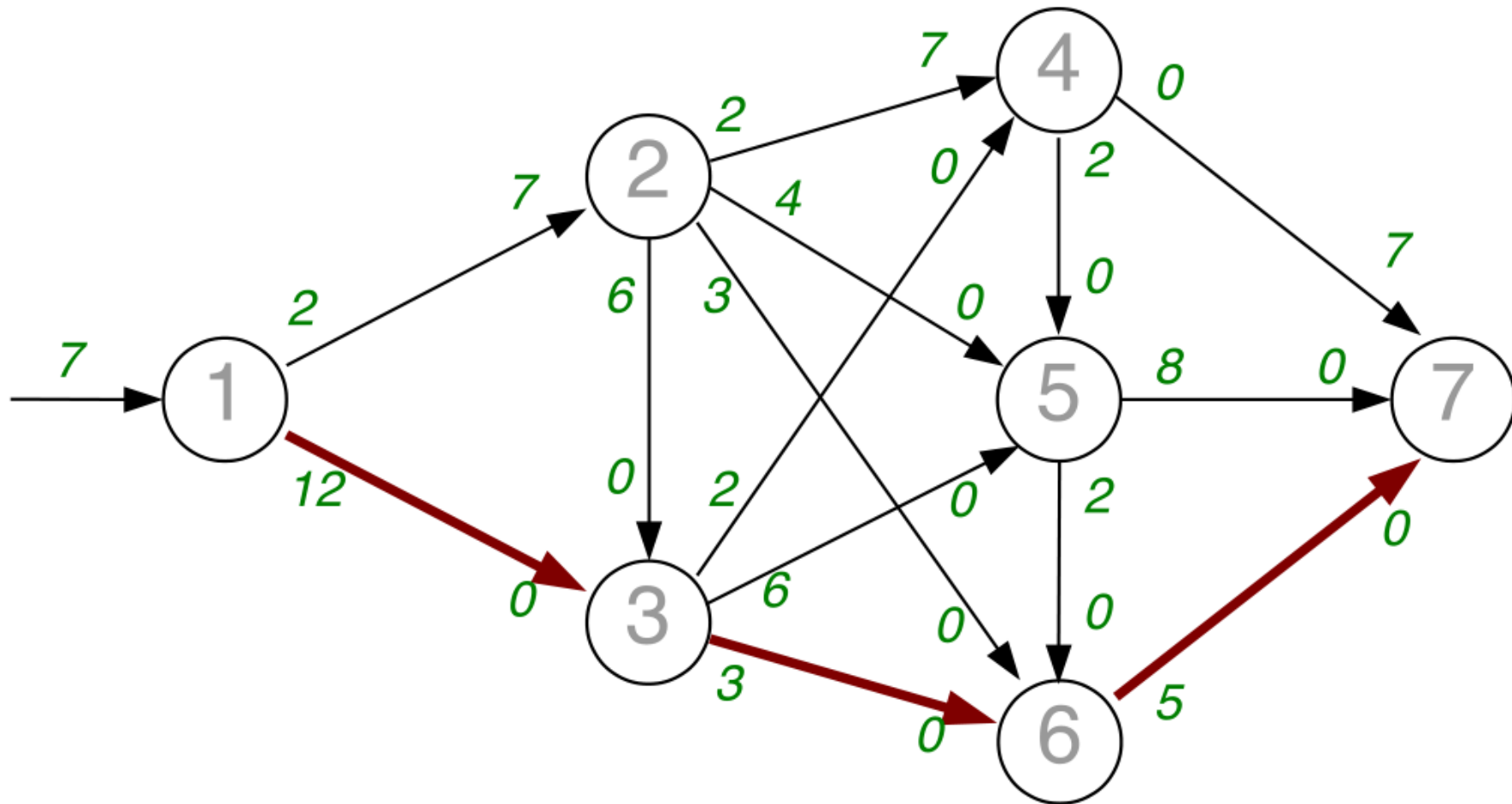
- Augmenting path $P: 1-2-4-7$
- Maximal flow increment: $\delta = 7$

Iteration 1



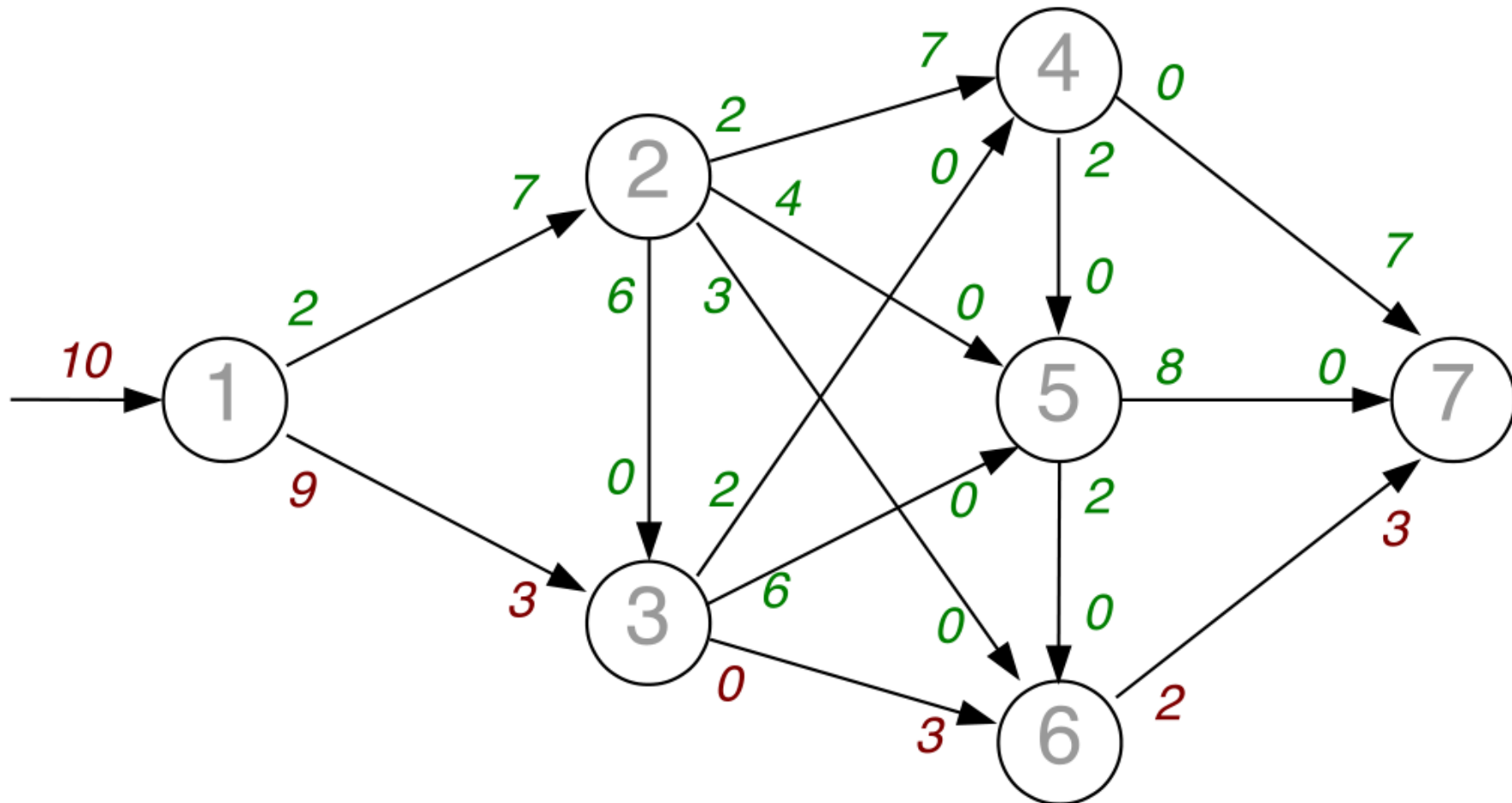
- Update of capacities along P and update of the flow value
- Remark: the edge $(4,7)$ disappears in the residual network

Iteration 2



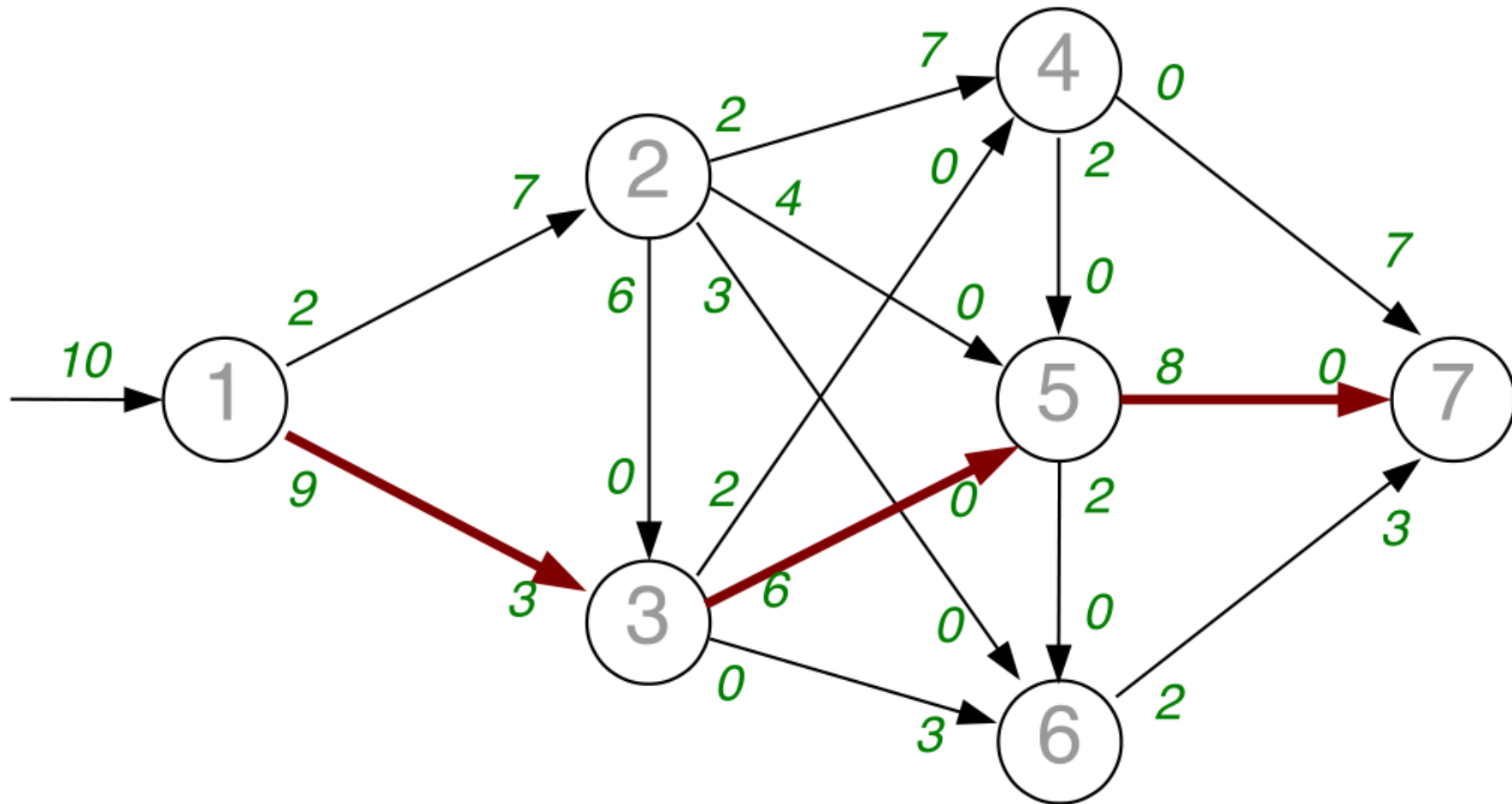
- Augmenting path $P: 1-3-6-7$
- Maximal flow increment: $\delta = 3$

Iteration 2



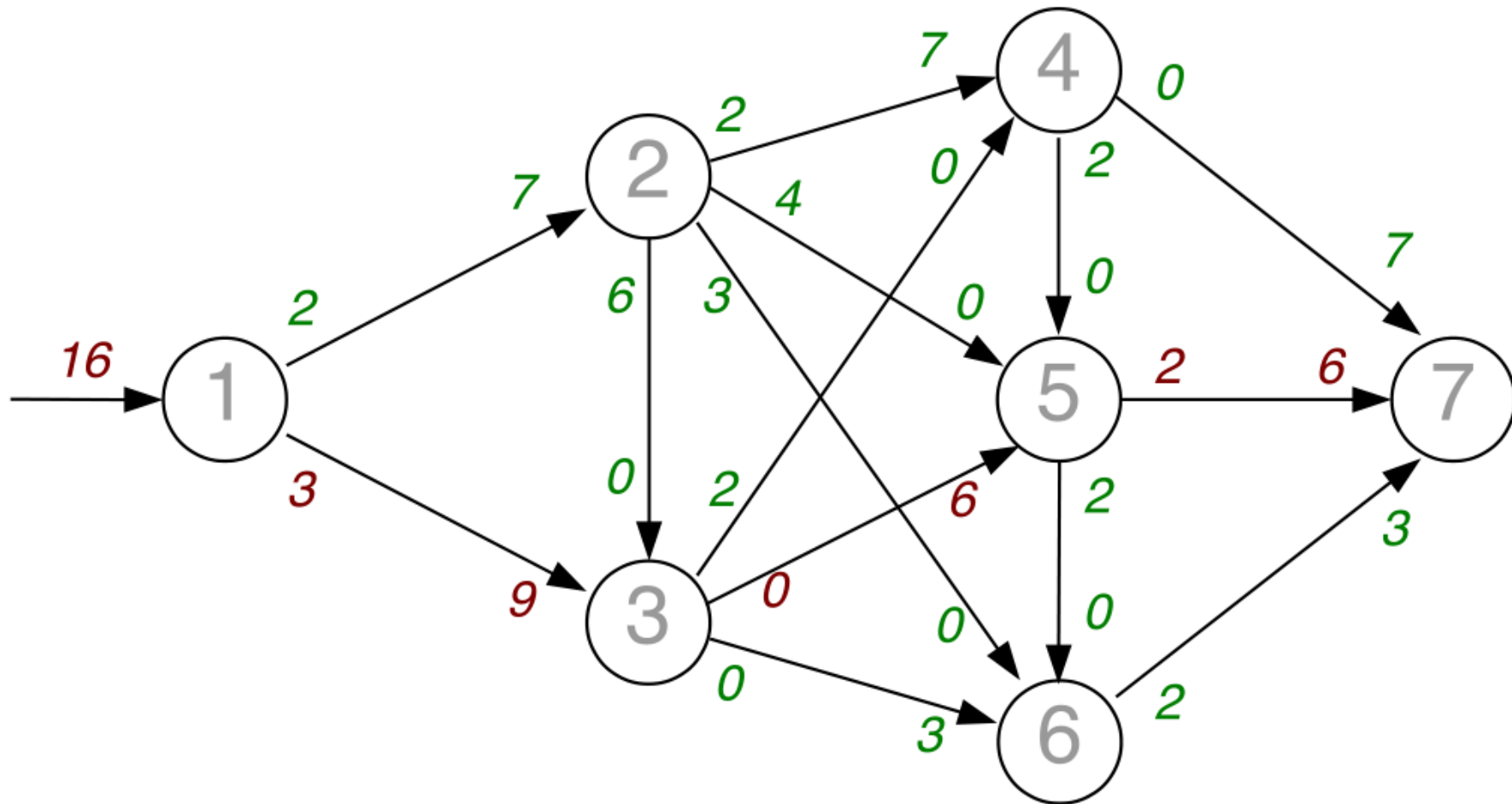
- Update of capacities along P and update of the flow value
- Remark: the edge $(3,6)$ disappears in the residual network

Iteration 3



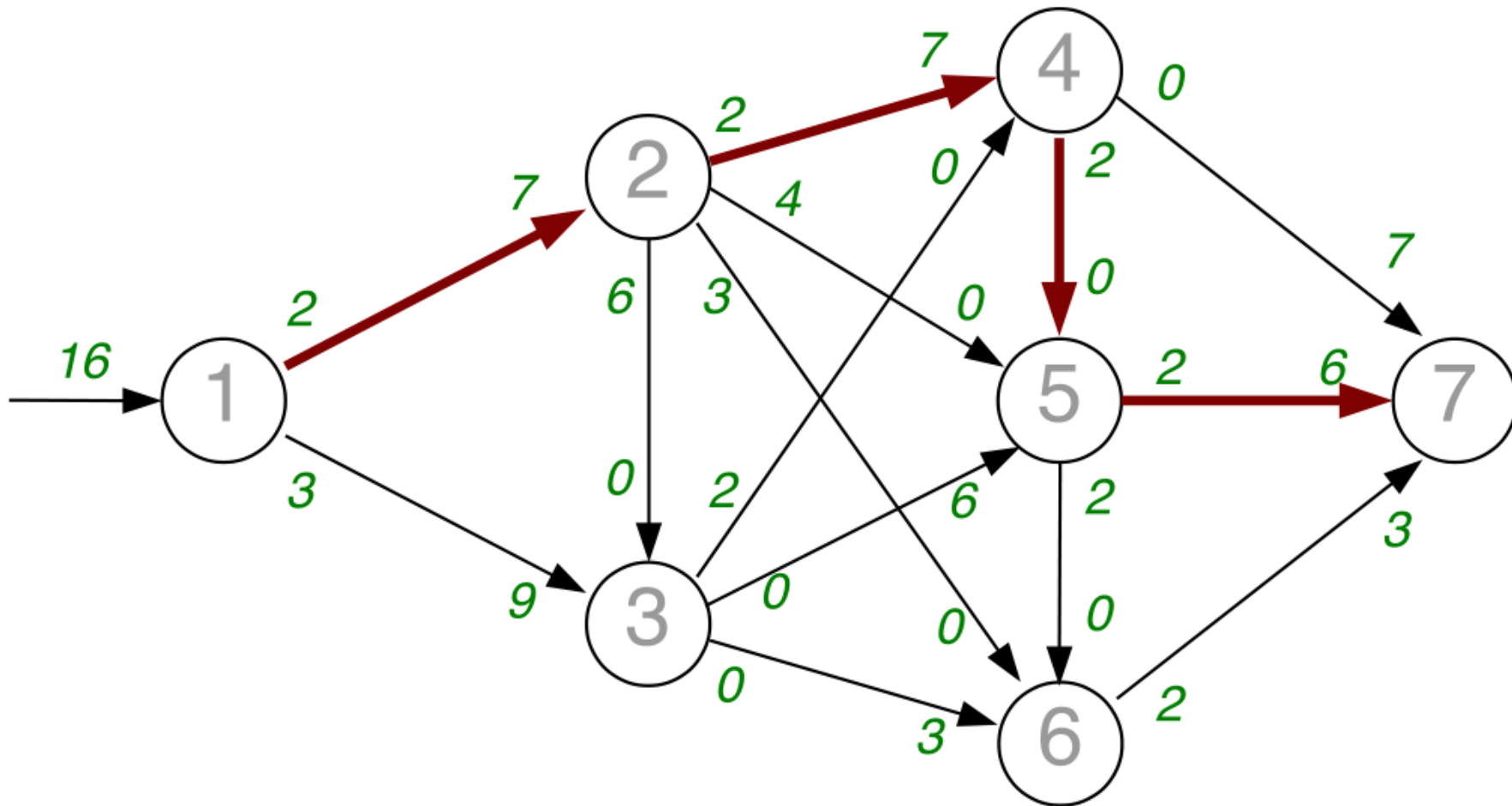
- Augmenting path $P: 1-3-5-7$
- Maximal flow increment: $\delta = 6$

Iteration 3



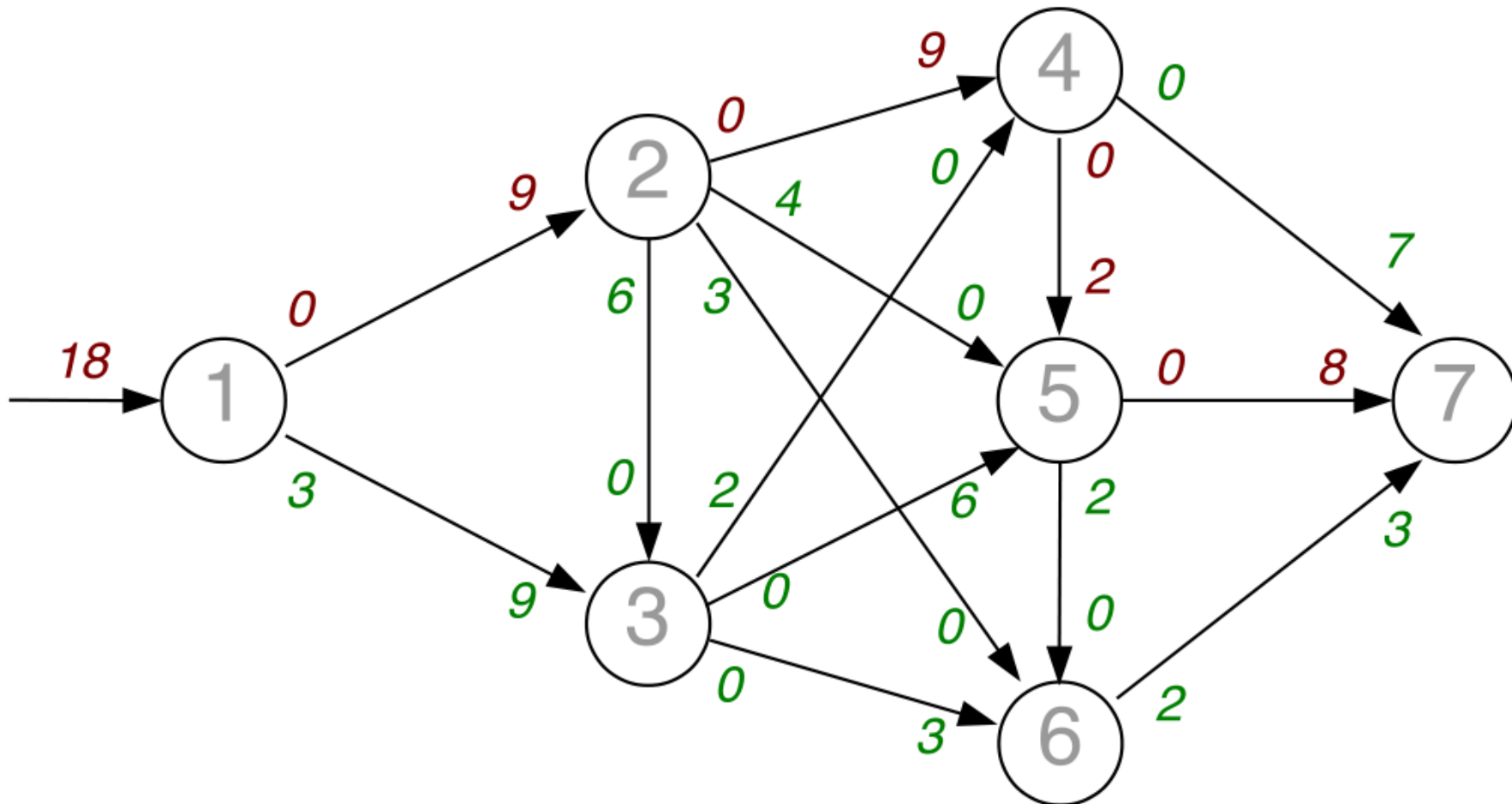
- Update of capacities along P and update of the flow value
- Remark: the edge (3,5) disappears in the residual network

Iteration 4



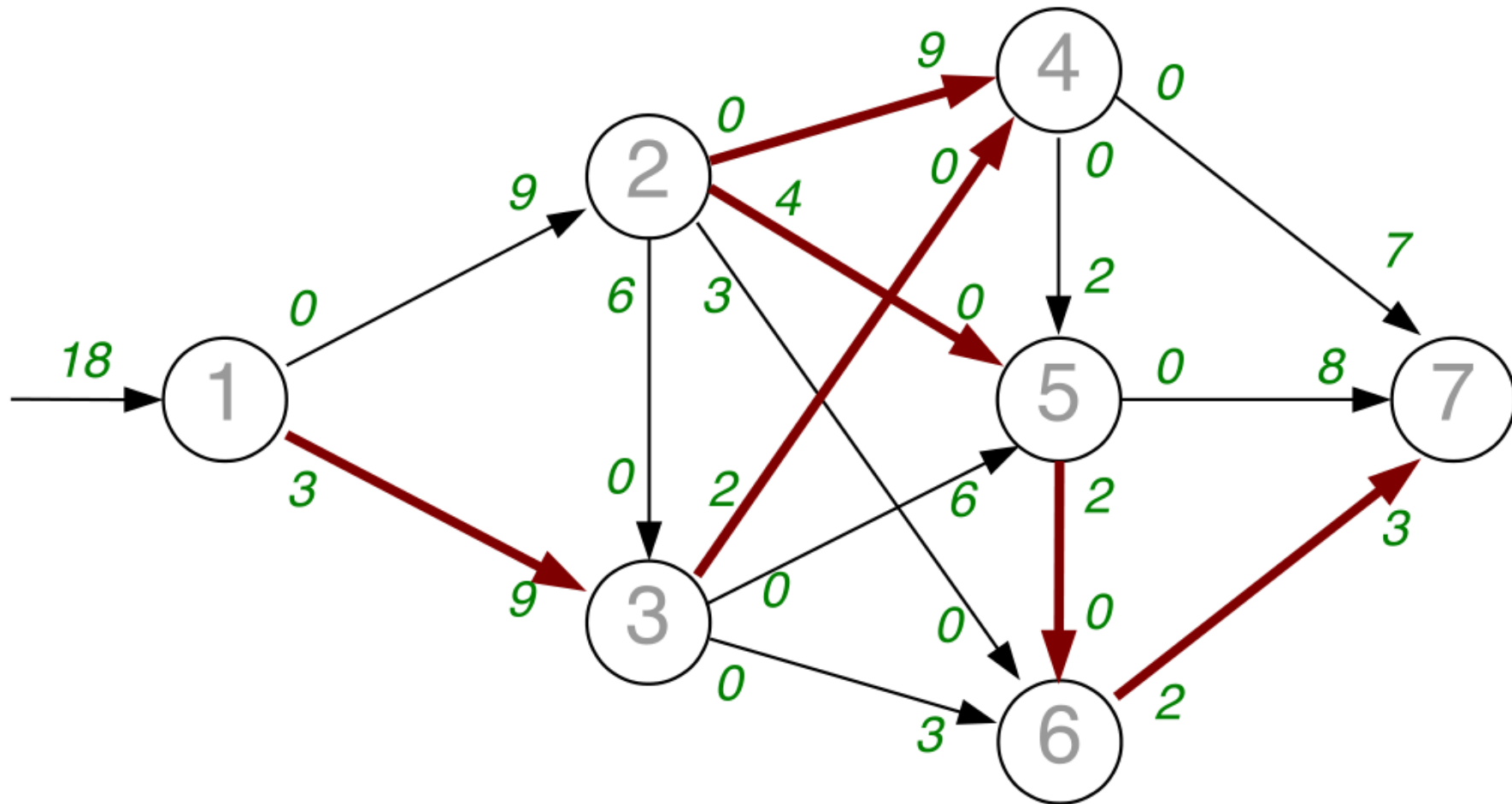
- Augmenting path $P: 1-2-4-5-7$
- Maximal flow increment: $\delta = 2$

Iterazione 4



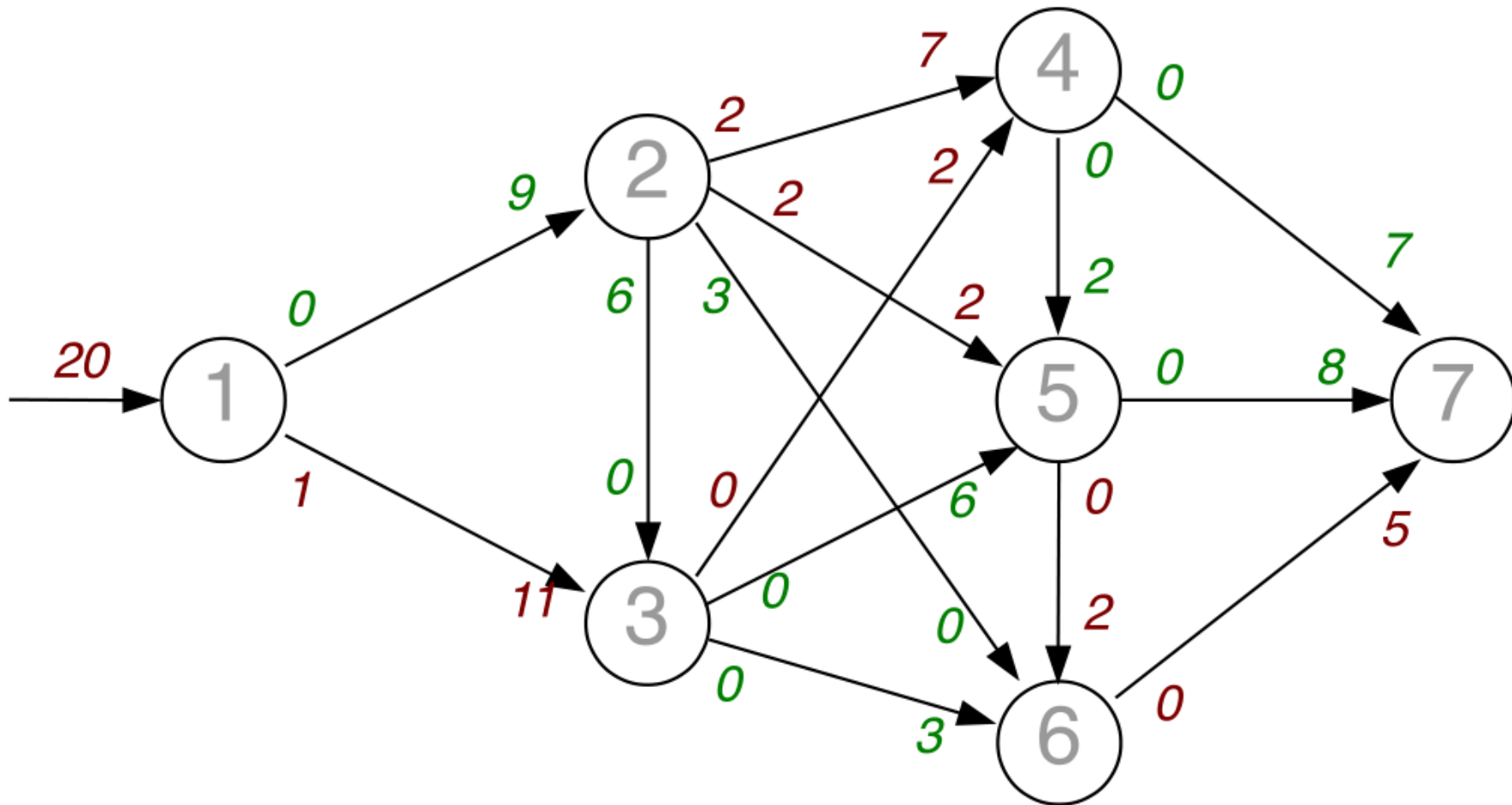
- Update of capacities along P and update of the flow value
- Remark: the edge (4,5) disappears in the residual network

Iteration 5



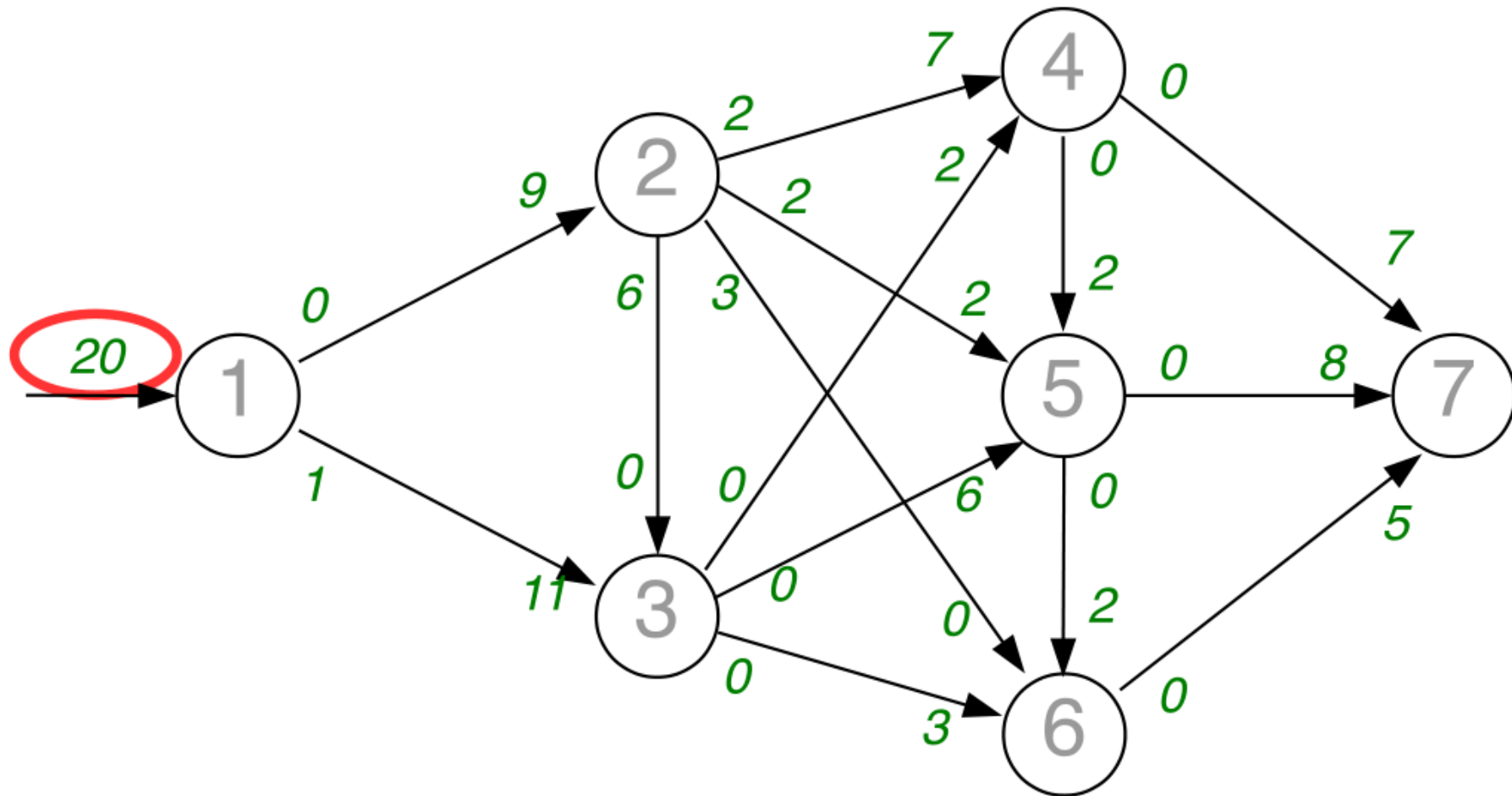
- Augmenting path $P: 1-3-4-2-5-6-7$
- Maximal flow increment: $\delta = 2$
- The inverse edge (4,2) has been used

Iteration 5



- Update of capacities along P and update of the flow value
- **Remark:** no additional flow can be sent from 4,5 and 6 to 7 \Rightarrow No augmenting path can exist even using inverse arcs

Iteration 5



The algorithm stops and the maximal flow value is 20