Mobile Robot Control by Dynamic Programming

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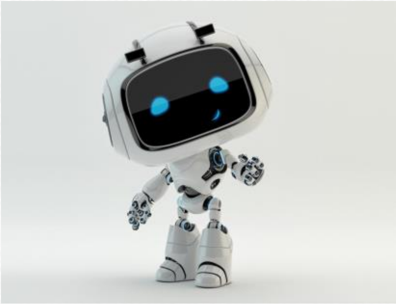
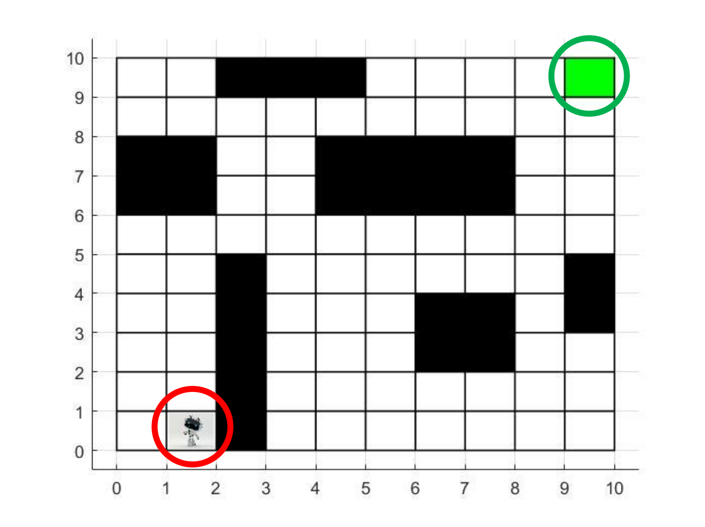
Mobile Robot Control

Let consider a mobile robot which, starting from an initial point x_0 in a map, has to reach a goal $\bar{x}$ avoiding some obstacles, that may be moving.



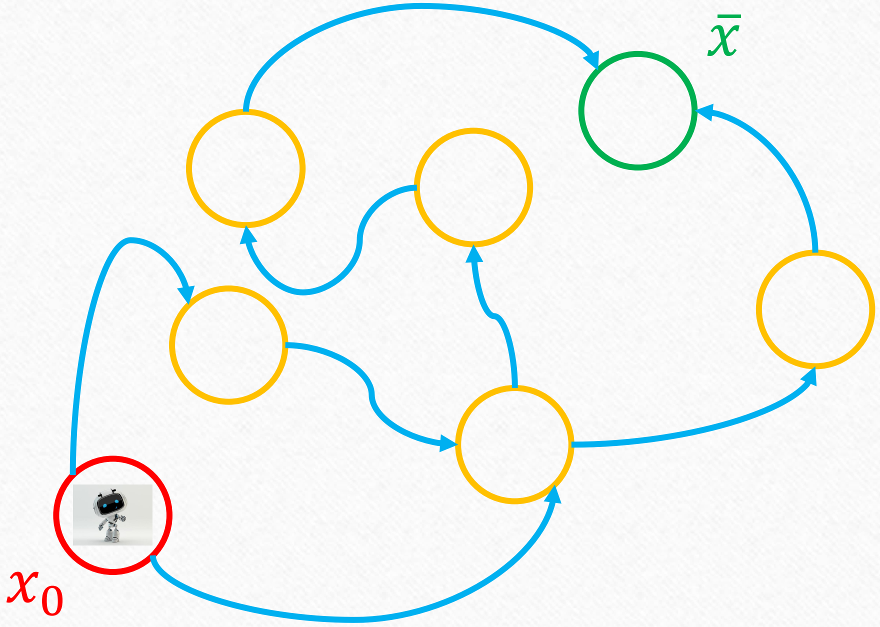


Mobile Robot Control

Robot possible positions are represented as nodes of a graph in which arcs correspond to possible movements.

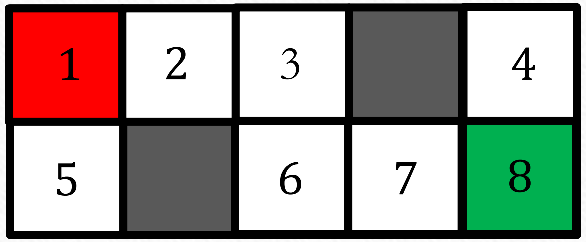
Given the robot position x, only a set $\mathcal{U}(x) \subseteq \mathcal{U}$ of actions may be accomplished by the robot, where

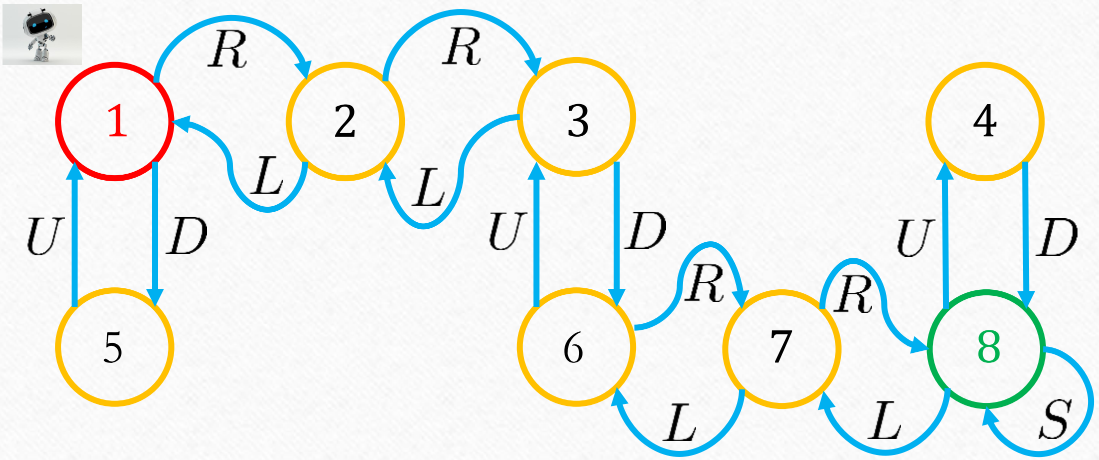
$$\mathcal{U} = \{UP, DOWN, LEFT, RIGHT, STOP\}$$



Numerical Example

A numerical example is here considered, in which the robot starts in $x = 1$ and by applying a sequence of actions has to reach $x = 8$





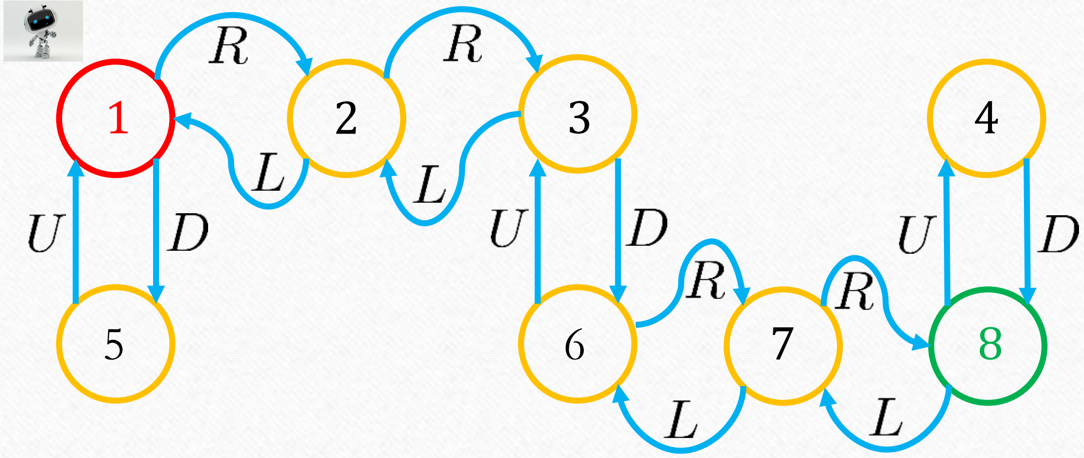
$$\mathcal{U} = \{UP, DOWN, LEFT, RIGHT, STOP\}$$

Numerical Example

Possible actions according to the different robot positions and their costs are given by

x	U	D	L	R	S
1	\	1	\	1	\
2	\	\	1	1	\
3	\	1	1	\	\
4	\	1	\	\	\
5	1	\	\	\	\
6	1	\	\	1	\
7	\	\	1	1	\
8	1	\	1	\	0

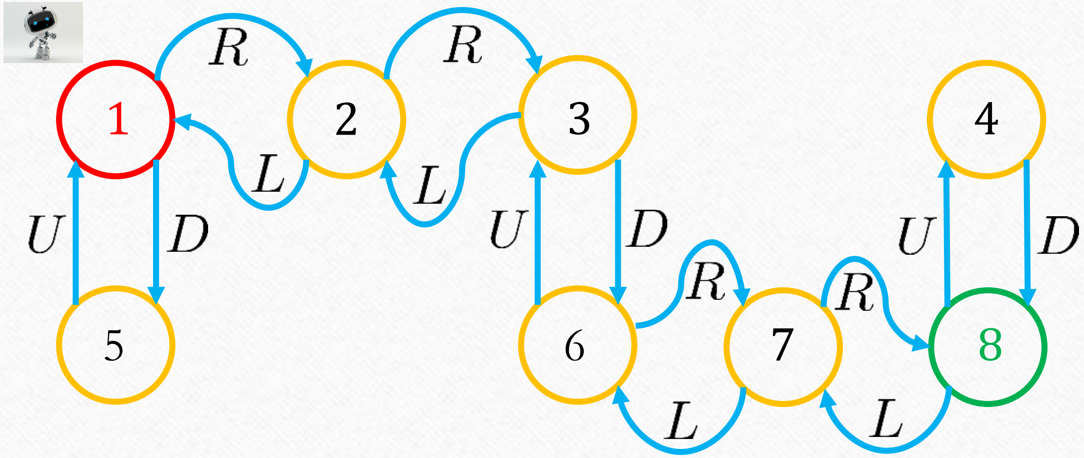
$$\mathcal{U}(x) \subseteq \mathcal{U}$$



Numerical Example

Consider a horizon $N = 6$. For the final cost we get the following table

x_6	$g(x_6)$
1	∞
2	∞
3	∞
4	1
5	∞
6	∞
7	1
8	0



Numerical Example

Now consider the final cost $J_6(x_6) = g(x_6)$. The computation of $J_5(x_5)$ and the state dependent optimal input $\mu_5(x_5)$ is obtained as follows

x_5	$J_5(x_5)$	$\mu_5(x_5)$
1	$\min\{(1+\infty),(1+\infty)\} = \infty$	R / D
2	$\min\{(1+\infty),(1+\infty)\} = \infty$	R / L
3	$\min\{(1+\infty),(1+\infty)\} = \infty$	L / D
4	$\min\{(1+0)\} = 1$	D
5	$\min\{(1+\infty)\} = \infty$	U
6	$\min\{(1+\infty),(1+1)\} = 2$	R
7	$\min\{(1+\infty),(1+0)\} = 1$	R
8	$\min\{(0+0),(1+1),(1+1)\} = 0$	S

Numerical Example

The computation of $J_4(x_4)$ and the state dependent optimal input $\mu_4(x_4)$ is obtained as follows

x_4	$J_4(x_4)$	$\mu_4(x_4)$
1	$\min\{(1+\infty),(1+\infty)\}=\infty$	R / D
2	$\min\{(1+\infty),(1+\infty)\}=\infty$	R / L
3	$\min\{(1+2),(1+\infty)\}=3$	D
4	$\min\{(1+0)\}=1$	D
5	$\min\{(1+\infty)\}=\infty$	U
6	$\min\{(1+\infty),(1+1)\}=2$	R
7	$\min\{(1+\infty),(1+0)\}=1$	R
8	$\min\{(0+0),(1+1),(1+1)\}=0$	S

Numerical Example

The computation of $J_3(x_3)$ and the state dependent optimal input $\mu_3(x_3)$ is obtained as follows

x_3	$J_3(x_3)$	$\mu_3(x_3)$
1	$\min\{(1+\infty),(1+\infty)\}=\infty$	R / D
2	$\min\{(1+3),(1+\infty)\}=4$	R
3	$\min\{(1+2),(1+\infty)\}=3$	D
4	$\min\{(1+0)\}=1$	D
5	$\min\{(1+\infty)\}=\infty$	U
6	$\min\{(1+3),(1+1)\}=2$	R
7	$\min\{(1+2),(1+0)\}=1$	R
8	$\min\{(0+0),(1+1),(1+1)\}=0$	S

Numerical Example

The computation of $J_2(x_2)$ and the state dependent optimal input $\mu_2(x_2)$ is obtained as follows

x_2	$J_2(x_2)$	$\mu_2(x_2)$
1	$\min\{(1+4),(1+\infty)\}=5$	R
2	$\min\{(1+3),(1+\infty)\}=4$	R
3	$\min\{(1+2),(1+4)\}=3$	D
4	$\min\{(1+0)\}=1$	D
5	$\min\{(1+\infty)\}=\infty$	U
6	$\min\{(1+3),(1+1)\}=2$	R
7	$\min\{(1+2),(1+0)\}=1$	R
8	$\min\{(0+0),(1+1),(1+1)\}=0$	S

Numerical Example

The computation of $J_1(x_1)$ and the state dependent optimal input $\mu_1(x_1)$ is obtained as follows

x_1	$J_1(x_1)$	$\mu_1(x_1)$
1	$\min\{(1+4),(1+\infty)\} = 5$	R
2	$\min\{(1+3),(1+5)\} = 4$	R
3	$\min\{(1+2),(1+4)\} = 3$	D
4	$\min\{(1+0)\} = 1$	D
5	$\min\{(1+5)\} = 6$	U
6	$\min\{(1+3),(1+1)\} = 2$	R
7	$\min\{(1+2),(1+0)\} = 1$	R
8	$\min\{(0+0),(1+1),(1+1)\} = 0$	S

Numerical Example

The computation of $J_0(x_0)$ and the state dependent optimal input $\mu_0(x_0)$ is obtained as follows

x_0	$J_0(x_0)$	$\mu_0(x_0)$
1	$\min\ \{(1+4), (1+6)\} = 5$	R
2	$\min\ \{(1+3), (1+5)\} = 4$	R
3	$\min\ \{(1+2), (1+4)\} = 3$	D
4	$\min\ \{(1+0)\} = 1$	D
5	$\min\ \{(1+5)\} = 6$	U
6	$\min\ \{(1+3), (1+1)\} = 2$	R
7	$\min\ \{(1+2), (1+0)\} = 1$	R
8	$\min\ \{(0+0), (1+1), (1+1)\} = 0$	S

Numerical Example

The optimal cost is obtained as $J(x_0) = J_0(x_0)$.

The state dependent optimal input sequence is given in the table, where x_i^* is the state obtained from x_{i-1}^* by applying the optimal input u_{i-1}^*, with $x_0^* = x_0$.

Optimal Input	
$\boldsymbol{u_0^*}$	$\mu(x_0)$
$\boldsymbol{u_1^*}$	$\mu(x_1^*)$
$\boldsymbol{u_2^*}$	$\mu(x_2^*)$
$\boldsymbol{u_3^*}$	$\mu(x_3^*)$
$\boldsymbol{u_4^*}$	$\mu(x_4^*)$
$\boldsymbol{u_5^*}$	$\mu(x_5^*)$