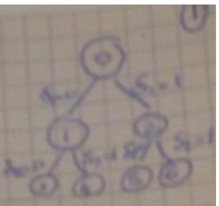
Ex 2 MILP

$$\max_{x_1,\delta_1,\delta_2} \ \delta_1 + \delta_2 - x_1$$
$$x_1 + \delta_1 \geq 1.5$$
$$\delta_1, \delta_2 \in \{0,1\}$$
$$x_1 \geq 0$$

Translation and relaxation node 0 $\longrightarrow$

$$\max \ \delta_1 + \delta_2 - x_1$$
$$x_1 + \delta_1 - s_1 = 1.5$$
$$\delta_1 + s_2 = 1$$
$$\delta_2 + s_3 = 1$$
$$x_1, s_1, s_2, s_3, \delta_1, \delta_2 \geq 0$$



Phase 1 relaxation node 0

$$\min \ y_1 + y_2 + y_3$$
$$x_1 + \delta_1 - s_1 + y_1 = 1.5$$
$$\delta_1 + s_2 + y_2 = 1$$
$$\delta_2 + s_3 + y_3 = 1$$
$$x_1, s_1, s_2, s_3, \delta_1, \delta_2, y_1, y_2, y_3 \geq 0$$

		x_1	s_1	s_2	s_3	δ_1	δ_2	y_1	y_2	y_3
$\rightarrow$	0	0	0	0	0	0	0	1	1	1
x_1	1.5	1	-1	0	0	1	0	1	0	0
s_2	1	0	0	1	0	1	0	0	1	0
s_3	1	0	0	0	1	0	0	0	0	1

Phase 1 OK! since I start having in the basis 3 variables which are not y_1, y_2, y_3 I can proceed with Phase 2. (feasibility is guaranteed)

If this does not happen at the beginning, I need to solve Phase 1 til the end.

Phase 2 for relaxation of node 0

		x_1	s_1	s_2	s_3	δ_1	δ_2
	0	-1	0	0	0	1	1
	1.5	1	-1	0	0	1	0
s_2	1	0	0	1	0	1	0
s_3	1	0	0	0	1	0	1

$\longrightarrow$ Add 2nd row to the 1st to bring x_1 in the basis

		x_1	s_1	s_2	s_3	δ_1	δ_2	
	1.5	0	-1	0	0	2	1	
x_1	1.5	1	-1	0	0	1	0	
s_2	[1	0	0	1	0	1	0]	AUX
s_3	1	0	0	0	1	0	1	

1.5/1 y_1

$-2 * AUX = [-2\ 0\ 0\ -2\ 0\ -2\ 0]$
$\downarrow$
sum this to 1st row
and sum $-AUX$ to 2nd row

		x_1	s_1	s_2	s_3	δ_1	δ_2	
	-0.5	0	-1	-2	0	0	1	$-[1\ 0\ 0\ 0\ 1\ 0\ 1]$
x_1	0.5	1	-1	-1	0	0	0	
δ_1	1	0	0	1	0	1	0	
s_3	[1	0	0	0	1	0	1]	AUX

$\downarrow$

		x_1	s_1	s_2	s_3	δ_1	δ_2
	-1.5	0	-1	-2	-1	0	0
x_1	0.5	1	-1	-1	0	0	0
δ_1	1	0	0	1	0	1	0
δ_2	1	0	0	0	1	0	1

Phase 2 finished!

$$x = \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{matrix} x_1 \\ s_1 \\ s_2 \\ s_3 \\ \delta_1 \\ \delta_2 \end{matrix}$$

$\rightarrow$ Since δ_1 and δ_2 have binary values the solution of the relaxed problem is the optimal solution of the original MILP!

Optimal cost 1.5