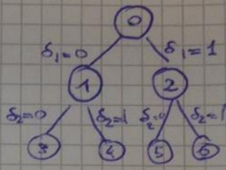
1

$$\max \quad 9x_1 + 5x_2 + 6\delta_1 + 4\delta_2$$
$$6x_1 + 3x_2 + 5\delta_1 + 2\delta_2 \leq 10$$
$$\delta_1, \delta_2 \in \{0, 1\}$$
$$x_1, x_2 \geq 0$$

$\downarrow$



$$\max \quad 9x_1 + 5x_2 + 6\delta_1 + 4\delta_2$$
$$6x_1 + 3x_2 + 5\delta_1 + 2\delta_2 + s_1 = 10$$
$$\delta_1, \delta_2 \in \{0, 1\}$$
$$x_1, x_2, s_1 \geq 0$$

Relaxation of node 0

$$\max \quad 9x_1 + 5x_2 + 6\delta_1 + 4\delta_2$$
$$6x_1 + 3x_2 + 5\delta_1 + 2\delta_2 + s_1 = 0$$
$$\delta_1 + s_2 = 1$$
$$\delta_2 + s_3 = 1$$
$$x_1, x_2, s_1, s_2, s_3, \delta_1, \delta_2 \geq 0$$

Phase 1

2

$$\min \quad y_1 + y_2 + y_3$$
$$6x_1 + 3x_2 + 5\delta_1 + 2\delta_2 + s_1 + y_1 = 10$$
$$\delta_1 + s_2 + y_2 = 1$$
$$\delta_2 + s_3 + y_3 = 1$$
$$x_1, x_2, s_1, s_2, s_3, \delta_1, \delta_2, y_1, y_2, y_3 \geq 0$$

		x_1	x_2	s_1	s_2	s_3	δ_1	δ_2	y_1	y_2	y_3
	0	0	0	0	0	0	0	0	1	1	1
s_1	10	6	3	1	0	0	5	2	1	0	0
s_2	1	0	0	0	1	0	1	0	0	1	0
s_3	1	0	0	0	0	1	0	1	0	0	1

OK Fase 1 → Phase 2

	x_1	x_2	s_1	s_2	s_3	δ_1	δ_2
0	9	5	0	0	0	6	4
10	6	3	1	0	0	5	2
1	0	0	0	1	0	1	0
1	0	0	0	0	1	0	1

$$AUX = \left[\, 5/3 \quad 1 \quad 1/2 \quad 1/6 \quad 0 \quad 0 \quad 5/6 \quad 1/3 \,\right]$$
$$- \left[\, 15 \quad 9 \quad 9/2 \quad 3/2 \quad 0 \quad 0 \quad 15/2 \quad 3 \,\right]$$

(3)

		x_1	x_2	s_1	s_2	s_3	δ_1	δ_2
	-15	0	$1/2$	$-3/2$	0	0	$-3/2$	1
x_1	$5/3$	1	$1/2$	$1/6$	0	0	$5/6$	$1/3$
s_2	1	0	0	0	1	0	1	0
s_3	1	0	0	0	0	1	0	1

$-[5/3 \ \ 1 \ \ 1/2 \ \ 1/6 \ \ 0 \ \ 0 \ \ 5/6 \ \ 1/3]$

AUX =

$[10/3 \ \ 2 \ \ 1 \ \ 1/3 \ \ 0 \ \ 0 \ \ 5/3 \ \ 2/3]$

		x_1	x_2	s_1	s_2	s_3	δ_1	δ_2
	$-50/3$	-1	0	$-10/6$	0	0	$-14/6$	$2/3$
x_2	$10/3$	2	1	$1/3$	0	0	$5/3$	$2/3$
s_2	1	0	0	0	1	0	1	0
s_3	1	0	0	0	0	1	0	1

$-[2/3 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ \frac{2}{3} \ \ 0 \ \ \frac{2}{3}]$

$-[2/3 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ \frac{2}{3} \ \ 0 \ \ 2/3]$

AUX

$\frac{10}{3} \cdot \frac{3}{2} = 5$

$1/1 = 1 \rightarrow$ tighter bound

(4)

		x_1	x_2	s_1	s_2	s_3	δ_1	δ_2
	$-\frac{52}{3}$	-1	0	$-\frac{5}{3}$	0	$-\frac{2}{3}$	$-\frac{7}{3}$	0
x_2	$8/3$	2	1	$1/3$	0	$-\frac{2}{3}$	$5/3$	0
s_2	1	0	0	0	1	0	1	0
δ_2	1	0	0	0	0	1	0	1

OK! All coefficients in the first row are negative (≤ 0) and the elements in the 1st column positive (≥ 0). Problem solved! Optimal cost: $\frac{52}{3}$

Optimizers:

$$
x = \begin{bmatrix} 0 \\ 8/3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}
\begin{array}{l} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \\ \delta_1 \\ \delta_2 \end{array}
\longrightarrow
\begin{array}{l} x_1 = 0 \\ x_2 = 8/3 \\ \\ \delta_1 = 0 \\ \\ \delta_2 = 1 \end{array}
$$

The solution of the relaxed problem has a binary value for δ_1 and δ_2

$\downarrow$

It is the optimal solution of the MILP!

DONE