Exam **January 20, 2017**

Consider the system

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + \frac{1}{3}x_1^3 - x_2 \end{cases}$$

where x_1 and x_2 are the state variables which depend on time and all the variables are scalar.

1. Classify the system (is it linear, nonlinear, autonomous, time-varying or time-invariant?)
2. Determine the equilibrium points of the system.
3. Linearize the system around an equilibrium point of your choice. Is the analysis of the linearized system meaningful to conclude about the stability of the equilibrium point of the nonlinear system? Which theorems can be used to carry out the analysis in a formal way?
4. Now consider a modified version of the system

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + \frac{1}{3}x_1^3 - x_2 + u \end{cases}$$

   Determine the amplitude of the control input $u(t) = -Ksign\{\sigma\}$, with $\sigma(t) = x_2 + \alpha x_1$, such that a sliding mode is enforced in a finite time on the sliding manifold $\sigma(t) = 0$.
5. Select a value of the design parameter α such that the controlled system in sliding mode is a LTI system with closed loop bandwidth $\geq 100\,[rad/s]$.