Exam **July 5, 2017**

Consider the system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - 2sin(x_2) + x_2 \end{cases}$$

where x_1 and x_2 are the state variables which depend on time and all the variables are scalar.

1. Classify the system (is it linear, nonlinear, autonomous, time-varying or time-invariant?)
2. Determine the equilibrium point of the system.
3. Linearize the system around the equilibrium point. Is the analysis of the eigenvalues of the linearized system meaningful to conclude about the stability of the equilibrium point of the nonlinear system?
4. Which theorem can be used to classify the equilibria of nonlinear systems as those of the corresponding linearized systems?
5. Now consider a modified version of the system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - 2sin(x_2) + x_2 + u \end{cases}$$

   Determine the amplitude of the control input $u(t) = -Ksign\{\sigma(t)\}$, with $\sigma(t) = x_2 + \alpha x_1$, such that a sliding mode is enforced in a finite time on the sliding manifold $\sigma(t) = 0$.
6. Select a value of the design parameter α such that the controlled system in sliding mode is an exponentially stable LTI system.