Course of Advanced Automation and Control

Exam for the students of the a.y. 2016/2017

February 9, 2017

Surname _____ Name _____

Part II - Nonlinear Control (Prof. A. Ferrara)

Consider the system

$$\begin{cases} \dot{x}_1 &= -x_2 \\ \dot{x}_2 &= x_1 + (x_1^2 - 1)x_2 \end{cases}$$

where x_1 and x_2 are the state variables which depend on time, and all the variables are scalar.

1. Classify the system (is it linear, nonlinear, autonomous, time-varying or time-invariant?)
2. Verify that the origin is an equilibrium point of the system.
3. Verify, by applying the linearization method, that such an equilibrium point is asymptotically stable.
4. Precisely state the Hartman-Grobman Theorem; check if its assumptions are satisfied in case of the considered system, and, if so, apply the theorem to classify the type of equilibrium the system exhibits at the origin.
5. Now consider a modified version of the system

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 + (x_1^2 - 1)x_2 + u \end{cases}$$

   and show that, by selecting $\sigma(t) = x_2 + \alpha x_1$ and the control u such that a sliding mode is enforced in a finite time on the sliding manifold $\sigma(t) = 0$, the controlled system in sliding mode (i.e. the "equivalent system")is a first order system.
6. Determine the interval of values of the design parameter α such that the "equivalent system" is exponentially stable.