Course of Advanced Automation and Control

Exam for the students of the a.y. 2016/2017

June 13, 2017

Surname _____ Name _____

Part II - Nonlinear Control (Prof. A. Ferrara)

Consider the system

$$\begin{cases} \dot{x}_1 &= -10x_2 \\ \dot{x}_2 &= 10x_1 - 101 \cdot \sin(x_2) \end{cases}$$

where x_1 and x_2 are the state variables which depend on time, and all the variables are scalar. Assume that $x_2 \in (-\pi/2, \pi/2)$.

1. Classify the system (is it linear, nonlinear, autonomous, time-varying or time-invariant?)
2. Verify that the origin is an equilibrium point of the system.
3. Verify, by applying the linearization method, that such an equilibrium point is asymptotically stable.
4. Precisely state the LaSalle Theorem. Is it possible to apply it to analyze the evolution in time of the considered system state? Motivate the answer.
5. Now consider a modified version of the system

   $$\begin{cases} \dot{x}_1 &= -10x_2 \\ \dot{x}_2 &= 10x_1 - 101 \cdot \sin(x_2) + u \end{cases}$$

   and show that, by selecting $\sigma(t) = x_2 - (\alpha/10)x_1$ and the control u such that a sliding mode is enforced in a finite time on the sliding manifold $\sigma(t) = 0$, the controlled system in sliding mode (i.e. the "equivalent system") is a first order system.
6. Determine the interval of values of the design parameter α such that the "equivalent system" is exponentially stable.