Course of Advanced Automation and Control

Exam for the students of the a.y. 2016/2017

September 22, 2017

Surname _____ Name _____

Part II - Nonlinear Control (Prof. A. Ferrara)

Consider the system

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2 \cdot sin(x_1) + (x_1^2 - 3)x_2 \end{cases}$$

where x_1 and x_2 are the state variables which depend on time, and all the variables are scalar.

1. Classify the system (is it linear, nonlinear, autonomous, time-varying or time-invariant?)
2. Determine how many equilibrium points the system has and select one of them.
3. By applying the linearization method, verify if it is possible to conclude about the stability property of the selected equilibrium point and, if possible, provide the conclusion.
4. Which is the aim of the Hartman-Grobman Theorem?
5. List the assumptions of the Hartman-Grobman Theorem and verify if they are satisfied for the considered system. If so, apply the theorem to classify the type of equilibrium the system exhibits at the origin.
6. Now consider a modified version of the system

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2 \cdot sin(x_1) + (x_1^2 - 3)x_2 + u \end{cases}$$

   and show that, by selecting $\sigma(t) = x_2 + 0.1x_1$ and the control u such that a sliding mode is enforced in a finite time on the sliding manifold $\sigma(t) = 0$, the controlled system in sliding mode (i.e. the "equivalent system") is a first order LTI autonomous system.
7. Write the expression of the free response of the state of the equivalent system to an initial condition $x_1(t_r)$ (t_r is the "reaching time", starting from which the sliding mode is enforced).