Course of Advanced Automation and Control

Exam for the students of the a.y. 2016/2017

September 8, 2017

Surname ______________________ Name ______________________

Part II - Nonlinear Control (Prof. A. Ferrara)

Consider the system

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1^2 - 2sin(x_2 - \pi/2) + x_2 \end{cases}$$

where x_1 and x_2 are the state variables which depend on time and all the variables are scalar.

1. Classify the system (is it linear, nonlinear, autonomous, time-varying or time-invariant?)
2. Determine the equilibrium states of the system.
3. Linearize the system around an equilibrium point with non negative coordinates.
4. Is the analysis of the eigenvalues of the linearized system meaningful to conclude about the stability of the equilibrium point of the nonlinear system?
5. How to proceed in general, when no conclusion can be drawn using the linearized system?
6. Provide the definition of *region of attraction* of an equilibrium state.
7. Now consider a modified version of the system

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2sin(x_2 - \pi/2) + 20u \end{cases}$$

   Determine the amplitude of the control input $u(t) = -Ksign\{\sigma(t)\}$, with $\sigma(t) = x_2 + \alpha x_1$, such that a sliding mode is enforced in a finite time on the sliding manifold $\sigma(t) = 0$.

8. Is it possible to select α such that the controlled system in sliding mode is an exponentially stable LTI system?