Course of Advanced Automation and Control

Exam for the students from the a.y. 2016/2017

April 06, 2018

Surname ______________________ Name ______________________

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

1. The Grecchi S.p.a. is a company manufacturing cars. At the moment the company focuses on three different car models (M1, M2, M3). The production of each car requires the purchase of raw material and three different phases: Phase 1 (manufacturing process), Phase 2 (assembling) and Phase 3 (quality control). The production costs for each car are reported in Table 1. As it can be noticed, Phase 2 can be performed either by human operators or by robots. The use of robots reduces dramatically the cost of Phase 2 but comes with a fixed cost of $250K\$$.

	Material Cost	Phase 1	Phase 2 (human operators)	Phase 2 (robots)	Phase 3	Selling Price	Car size
M1	10 $K\$$	2 $K\$$	5 $K\$$	0.5 $K\$$	5 $K\$$	35 $K\$$	5 m^2
M2	15 $K\$$	3 $K\$$	7 $K\$$	0.7 $K\$$	10 $K\$$	50 $K\$$	7 m^2
M3	20 $K\$$	4 $K\$$	9 $K\$$	0.9 $K\$$	20 $K\$$	70 $K\$$	9 m^2

Table 1

The Grecchi S.p.a. is interested in maximizing profit ($revenue - cost$) over the next 5 years and needs to decide how many cars to produce of each model and whether to acquire robots or not. Besides, if the monthly production (total production of $M1 + M2 + M3$ divided by 60, number of months in 5 years) exceeds the 100 units, the company needs to buy new land and build new warehouses for storing the cars. Each new warehouse has a cost of $100K\$$ and a size of $300m^2$. The company needs to decide whether to produce more than 100 units per month and, if so, depending on how many cars are produced and depending on their size, how many warehouses to build.

Please formulate the problem above as a mixed integer linear program to support the decision-making process of Mr. Grecchi (assume that all the produced cars are sold).

Very important note: while formulating the problem above, you will obtain bilinear terms like $M_i\delta_i$ with M_i indicating the number of produced cars of a certain model and δ_i a binary variables associated to the purchase of robots. A similar thing will occur between a *delta* indicating the need of building warehouses and the number of warehouses to be built. In order the problem to be an MILP, such terms need to disappear from the problem and replaced by new variables y_i subject to the following constraints:

$$y_i \leq M\delta_i \quad y_i \geq m\delta_i \quad y_i \leq M_i - m(1-\delta_i) \quad y_i \geq M_i - M(1-\delta_i)$$

with $M = \max(M_i)$ and $m = \min(M_i)$. Assume that each M_i is upper bounded by 10^5 and an upper bound on the number of warehouses is 200.

2. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{array}{lll} \max_{x_1,\delta_1,\delta_2} & x_1 + 2\delta_1 + 4\delta_2 & \\ & x_1 + \delta_1 & \leq 0.5 \\ & \delta_1, \delta_2 \in \{0,1\} & \\ & x_1 \geq 0 & \end{array}$$

3. For the directed network in the figure below, compute all shortest paths from vertex 5 to all other vertices.