Course of Advanced Automation and Control

Exam for the students of the a.y. 2016/2017

February 09, 2017

Surname ____________________ Name ____________________

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

1. Mr. Grecchi loves fishing and decided to convert his hobby in a real job. First of all he needs a boat. He identified two types of boats: boat A and boat B (he can buy only one boat). The specs of the boats are provided in Table 1. Moreover, two types of fishing equipments are considered: equipment A (EA) and equipment B (EB). Their costs and efficacy (in terms of fish caught per hour) are reported in Table 2.

	Cost	Fuel Consumption
Boat A	80000 *Euro*	$36L/h$
Boat B	150000 *Euro*	$70L/h$

Table 1

	Cost	Efficacy
EA	5000 *Euro*	$9Kg/h$
EB	10000 *Euro*	$20Kg/h$

Table 2

Note that, due to space limitations, equipment B can be used only on Boat B. In any case, Mr. Grecchi can buy only one piece of equipment (either EA or EB). The objective of Mr. Grecchi is to evaluate his profit over 5 years (at most 8000 hours of effective work). Take into account that the fuel cost is $0.8€/L$ and that the fish selling cost is $5€/Kg$. Please formulate the problem above as a mixed integer linear program to support the decision-making process of Mr. Grecchi (assume that all the caught fish is sold).

Very important note: while formulating the problem above, you will obtain bilinear terms like $n_h\delta_i$ with n_h indicating the number of fishing hours and δ_i a binary variables associated to the choice of boat or equipment. In order the problem to be an MILP, such terms need to disappear from the problem and be replaced by new variables y_i subject to the following constraints:

$$y_i \leq M\delta_i \quad y_i \geq m\delta_i \quad y_i \leq n_h - m(1-\delta_i) \quad y_i \geq n_h - M(1-\delta_i)$$

with $M = \max(n_h)$ and $m = \min(n_h)$.

2. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{array}{lll} \max_{x_1,\delta_1,\delta_2} & x_1 + 2\delta_1 + 4\delta_2 & \\ & x_1 + \delta_1 & \leq 0.5 \\ & \delta_1, \delta_2 \in \{0,1\} & \\ & x_1 \geq 0 & \end{array}$$

3. For the directed network in the figure below, compute all shortest paths from vertex 5 to all other vertices.