Course of Advanced Automation and Control

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Surname ____________________ Name ____________________

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

1. Mr. Grecchi is considering the possibility of starting a car sharing company in Milan. In this business, the renting price has a fixed component (p_f) and a variable one. This latter depends on the duration of the rent. The total renting price is therefore $p = p_f + p_v * m$ with p_v the variable part of the price and m the number of minutes the car has been rented for. The analysts of Mr. Grecchi have estimated that the customers demand would depend on the renting price as described in Table 1.

Case	Conditions	Maximum number of contemporary requests	Estimated number of requests/day	average duration of the rent
1	$p_f \leq 8$€ and $p_v \leq 0.2$€/min	250	2000	$75min$
2	$p_f > 8$€ and $p_v \leq 0.2$€/min	200	1500	$75min$
3	$p_f \leq 8$€ and $p_v > 0.2$€/min	150	1600	$55min$
4	$p_f > 8$€ and $p_v > 0.2$€/min	100	1000	$50min$

Table 1

Mr. Grecchi has to decide:

(a) The number of cars to buy for his rental fleet. Each car has a cost of $40K$€. Note that the size of the fleet needs to satisfy the maximum number of contemporary requests (if one can have 200 contemporary requests, then the fleet should have at least 200 cars).

(b) The fixed price p_f which in any case should satisfy the following constraint: $5 \leq p_f \leq 10$.

(c) The variable price p_v which in any case should satisfy the following constraint: $0.1 \leq p_v \leq 0.25$.

The daily revenue can be computed by multiplying the renting price (p_v gets multiplied by the average duration of the rent) by the number of estimated requests/day. The objective of Mr. Grecchi is to optimally choose the value of the variables above in order to maximise his profit over the next 5 years (1826 days). Note that besides the cost of buying the cars, Mr. Grecchi will have to consider a variable cost associated to the car usage, which is approximated as 0.05€/min.

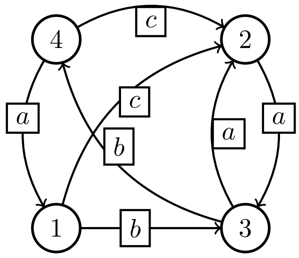
Please formulate the problem above as a MILP to support the decision-making process of Mr. Grecchi.

Suggestions: Define separate binary variables for the inequality conditions on p_f and p_v. Then, use extra binary variables to describe the cases $(1, 2, 3, 4)$ and link them to the other binaries through inequality conditions. Once this is done, you still will have to deal with bilinearities between prices and this last set of binaries.

2. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{aligned} \min_{x_1,\delta_1,\delta_2} \quad & -\delta_1 - \delta_2 + x_1 \\ & \delta_1 \leq -0.5\delta_2 + x_1 \\ & \delta_1, \delta_2 \in \{0, 1\} \end{aligned}$$

3. Consider the automaton in the figure ($C = \{a, b, c\}$ is the set of control values and $S = \{1, 2, 3, 4\}$ is the set of state values) with the intermediate cost $g(x, u)$ and the terminal cost $g_3(x)$ given below



$g(x,u)$	a	b	c
1	-	1	1.5
2	1	-	-
3	0.4	3	-
4	2	-	0.5

$$g_3(x) = \begin{cases} 3 & \text{if } x = 1 \\ 0 & \text{if } x = 2 \\ 1 & \text{if } x = 3 \\ 2 & \text{if } x = 4 \end{cases}$$

2.1 Solve the optimal control problem

$$J(x_0) = \min_{u_0,u_1,u_2} g_3(x_3) + \sum_{k=0}^{2} g(x_k, u_k)$$

using dynamic programming.

2.2 Compute an optimal control sequence for $x_0 = 1$ and compute the optimal cost value.

4. For the directed network in the figure below, compute all shortest paths from vertex 4 to all other vertices.