Course of Advanced Automation and Control

September 14, 2018

Surname ______________________ Name ______________________

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

1. Mr. Peach is studying Engineering at University of Pavia. During the exam session of September he would like to give three exams: Industrial Control (6 Credits), Advanced Automation and Control (9 Credits) and Industrial Informatics and Embedded Systems (6 Credits) which we will indicate respectively as E1, E2 and E3. The exams are scheduled as follows: E1 - 07/09/2018, E2, 14/09/2018, E3, 21/09/2018. Assume Mr. Peach starts studying the 1st of September and dedicates 8 hours a day to the study, except for the days he gives an exam. **If**, for example, he gives E2, then he will study 0 hours that day. Let us divide the 20 days (01/09 to 20/09, 160 hours in total) into three parts: (P1: 01-06, P2: 07-13, P3: 14-20). During P1, Mr. Peach could divide his studying time between E1, E2, E3. During P2, between E2 and E3, etc. In order to pass an exam, he has to study at least 44 hours for E1, 52 for E2, 49 for E3. The objective of Mr. Peach is to minimize the number of studying hours that guarantees him to pass at least two exams and collect at least 15 Credits. Is it possible to pass all the three exams?

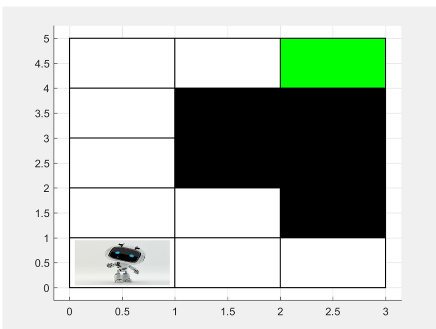
   Please formulate the problem above as a MILP to support the decision-making of Mr. Peach.

2. Please solve the following MILP problem using the branch and bound algorithm

$$
\begin{aligned}
\max_{x_1,\delta_1,\delta_2} \quad & 0.5x_1 + 1.5\delta_1 + 0.5\delta_2 \\
& -\delta_2 - 2x_1 \geq -1 + 2\delta_1 \\
& \delta_1, \delta_2 \in \{0, 1\} \\
& x_1 \leq 0
\end{aligned}
$$

3. Consider the mobile robot problem in the figure below. The square in the top right indicates the goal while the black squares the obstacles. Assume the robot can move {up,down,left,right} **but also diagonally**, if the diagonal movement does not cross a black square. Assume that the different actions have the following cost: {up,down,left,right}: +1, diagonal: +1.5. Assume also that an extra action {stay} is available at the goal only with cost 0.

   - For horizon $N = 8$, please formulate the dynamic program (write down the stage cost and the terminal cost) and solve the first two steps only.



4. For the directed network in the figure below, compute all shortest paths from vertex 2 to all other vertices. Moreover, assume you have the chance to add an edge of cost 1 between two nodes. Where would you add it to make the node that was not reachable from vertex 2, now reachable with a minimum cost?