Course of Advanced Automation and Control

June 19, 2018

Surname ____________________ Name ____________________

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

1. Mr. Grecchi is a car producer interested in improving his business. The local car market currently demands the production of 5000 cars/year. Mr. Grecchi shares the market with his historical rival Mr. Arcuri. Currently they produce 50% of the demanded cars each, keeping the car price at 20000€ a piece. The production cost is at the moment 10000€ per car. The construction of each car demands 10 man-hours. Mr. Grecchi has a total pool of 40000 man-hours/year.

   Mr. Grecchi is considering several options for increasing his profit over the next 5 years:

   a) keep the current strategy;

   b) invest in advertisement. Cost of the operation: 30M€. Increase of demand for Mr. Grecchi: 15%;

   c) stop outsourcing the car assembling. This action would reduce the production costs by 20% but would require the installation of a new facility for managing the operation. Fixed cost 20M€. Moreover, this option would require a fixed allocation of personnel for 5000man-hours/year and an increase of man-hours per car by 10%.

   d) Options b) and c) together.

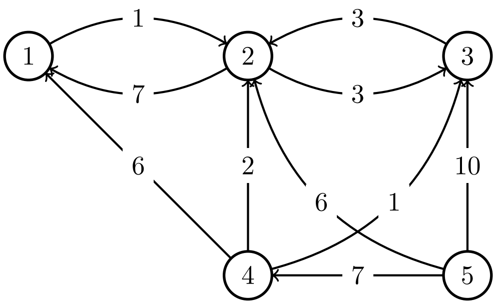
   The antitrust rules try to balance the market to avoid a monopoly. In particular, if a company produces more than 60% of the demanded cars, it has to pay to the government 2000€/car. Note that Mr. Grecchi can decide to produce less cars if needed (i.e. if he keeps the current strategy, he can still produce less than 50%. Also, if the options lead to more than 60%, he can still produce less). Please formulate the problem above as a MILP to support the decision-making process of Mr. Grecchi (assume all cars are sold).

   Very important note: while formulating the problem above, you will obtain bilinear terms like $x\delta_i$ with x indicating the number of produced cars and δ_i a binary variable. In order the problem to be an MILP, such terms need to disappear from the problem and replaced by new variables y_i subject to the following constraints: $y_i \leq M\delta_i, y_i \geq m\delta_i, y_i \leq x - m(1-\delta_i), y_i \geq x - M(1-\delta_i)$, with $M = \max(x)$ and $m = \min(x)$.

2. Please solve the following MILP problem using the branch and bound algorithm

$$
\begin{array}{lll}
\max_{x_1,\delta_1,\delta_2} & x_1 + 3\delta_1 + \delta_2 & \\
 & -0.5\delta_2 - x_1 - \delta_1 & \geq -0.5 \\
 & \delta_1, \delta_2 \in \{0, 1\} & \\
 & x_1 \leq 0 &
\end{array}
$$

3. For the directed network in the figure below, compute all shortest paths from vertex 5 to all other vertices.



4. Consider the mobile robot problem in the figure below. The square in the top right indicates the goal while the black squares the obstacles. Assume the robot can move {up,down,left,right} (no diagonal movements) and that each action has a cost of +1. Assume also that an extra action {stay} is available at the goal only with cost 0.

- What is the minimum horizon required to guarantee the attainment of the goal from any initial condition?
- For the obtained horizon please formulate the dynamic program (write down the stage cost and the terminal cost) and solve the first step (J_{N-1} and μ_{N-1}) only.