Course of Advanced Automation and Control

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Surname ______________________ Name ______________________

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

1. Mr. Larry is a vacuum cleaner representative which aims to maximize his daily profit by selling as many machines as possible. For this reason, Mr. Larry works 8 hours per day to get in contact with new customers using different strategies such as phone calls, e-mails and door-to-door sales. Each phone call costs 0.08$ and lasts for 5 minutes, while each e-mail costs 0.01$ and takes 30 seconds to be written. Finally, in order to reach customers house for door-to-door sale, Mr. Larry has to take the underground since he does not have the driving license. The door-to-door sale together with the moving requires 30 minutes. Mr. Larry can choose to buy a single underground ticket which lasts for 1 hour and costs 1.5$, or a daily ticket for 10$. In case he spends outside more than 4 hours he has to spend 25$ for lunch.

   Historical data show the following revenues:

   - 1 small vacuum cleaner ($130 each) sold every 100 e-mails.
   - 5 large vacuum cleaner ($200 each) and 7 small ones sold every 100 phone calls.
   - 2 large vacuum cleaner and 3 small ones sold every 10 door-to-door visits.

   Please formulate the problem above as a MILP to maximize the daily profit of Mr. Larry.

   Very important note: while formulating the problem above, you will obtain bilinear terms like $x\delta_i$. In order the problem to be an MILP, such terms need to disappear from the problem and replaced by new variables y_i subject to the following constraints: $y_i \leq M\delta_i, y_i \geq m\delta_i, y_i \leq x - m(1-\delta_i), y_i \geq x - M(1-\delta_i)$, with $M = \max(x)$ and $m = \min(x)$.

2. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{array}{ll} \min_{x_1,\delta_1,\delta_2} & \delta_1 + \delta_2 + x_1 - 2 \\ & \delta_2 + \delta_1 + 1 \geq x_1 + 1.4 \\ & \delta_1, \delta_2 \in \{0,1\} \\ & x_1 \geq 0 \end{array}$$

   What is the value of the optimal cost?

3. Consider a mobile robot problem involving **two robots** moving in the environment depicted below. The squares having coordinates $(0.5, 3.5)$ and $(1.5, 2.5)$ represent the goals of robot 1 and 2. The black squares indicate the obstacles. Assume the robot can move {up,down,left,right,stay} **only** and that the stay action has cost 0.1 while all other actions have cost 1. Keep also in mind that configurations of robots where they share a square **are not allowed**.

   - What is the minimum horizon required to guarantee the attainment of the goals from any allowed initial conditions?
   - Let $N = 8$. Please formulate the dynamic program (write down the stage cost and the terminal cost) and solve the first step (J_{N-1} and μ_{N-1}) only. Please do the computation only for the following positions: (0.5,3.5), (0.5,2.5), (1.5,2.5) and (2.5,4.5).