25-1-2019

Ex 1 $\quad \delta_1 = 1$ standard contract for employee 1

$\delta_2 = 1$ " " " " 2

$h_{1,2}, h_{1,3}, h_{1,4}, h_{1,5}, h_{1,6}$ = hours worked by employee 1 during 2^{nd}-6th day of the week

$h_{2,2}, \dots h_{2,6}$ = " employee 2

$\delta_{1,2}, \delta_{1,6} = 1$ if the employee 1 is working with flexible contract for a given (2^{nd}-6th) day of the week

$\delta_{2,2}, \dots \delta_{2,6} = 1$ " " employee 2 " " "

$$\min \quad 40 \cdot 16\,\delta_1 + 40 \cdot 16 \cdot \delta_2 + (1-\delta_1) \cdot \Big[18(h_{1,2}+h_{1,3}+h_{1,4}+h_{1,5}+h_{1,6}) + 5(\delta_{1,2}+\delta_{1,3}+\delta_{1,4}+\delta_{1,5}+\delta_{1,6}) \Big] +$$
$$+ (1-\delta_2) \cdot \Big[18(h_{2,2}+h_{2,3}+h_{2,4}+h_{2,5}+h_{2,6}) + 5(\delta_{2,2}+\delta_{2,3}+\delta_{2,4}+\delta_{2,5}+\delta_{2,6}) \Big]$$

subject to (*)

$$4\delta_{1,2} \le h_{1,2} \le 10 \qquad 4\delta_{2,2} \le h_{2,2} \le 10$$
$$4\delta_{1,3} \le h_{1,3} \le 10 \qquad 4\delta_{2,3} \le h_{2,3} \le 10$$
$$4\delta_{1,4} \le h_{1,4} \le 10 \qquad 4\delta_{2,4} \le h_{2,4} \le 10$$
$$4\delta_{1,5} \le h_{1,5} \le 10 \qquad 4\delta_{2,5} \le h_{2,5} \le 10$$
$$4\delta_{1,6} \le h_{1,6} \le 10 \qquad 4\delta_{2,6} \le h_{2,6} \le 10$$

$$30 \le \sum_{i=2}^{6} h_{1,i} \le 50 \qquad 30 \le \sum_{i=2}^{6} h_{2,i} \le 50$$

$$h_{1,2}+h_{2,2} \ge 7, \quad h_{1,3}+h_{2,3} \ge 10, \quad h_{1,4}+h_{2,4} \ge 12, \quad h_{1,5}+h_{2,5} \ge 13$$
$$h_{1,6}+h_{2,6} \ge 16$$

The cost function presents bilinearities (e.g. $\delta_1 \cdot h_{1,2}$ and $\delta_1 \cdot \delta_{1,2}$ ..) These should be replaced by introducing extra variables (e.g. $z_{1,2}$ and $\delta_{1..1,2..}$) Note also that the continuous variables are actually integer only since we assume an integer number of hours.

Further note: the constraints at (*) are not sufficient to force that $\delta_{12}=1$ when $h_{12} \ge 4$. Indeed h_{12} can be ≥ 4 even when $\delta_{12}=0$. Actually the optimization will set $\delta_{12}=0$ so to avoid to pay the token of 5€. To solve this issue we need to add $\delta_{12} = 1 \leftrightarrow h_{12} > 0$ [the same should be done for all δ_{ij}] and translate it in the corresponding constraints.