Industrial Automation - Advanced Automation and Control

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18 February 2011

Industrial Automation

1. A pharmaceutical company mixes three chemicals (C_1, C_2, C_3) to obtain cough tablets of two types: A and B. The availability and cost of each chemical are given in the following table.

	C_1	C_2	C_3
Availability (gr)	400	100	300
Cost (Euro /gr)	0.1	0.5	0.4

   Tablets must fulfill the composition constraints in the table below on the percentage of each chemical used in the production process.

	% of C_1	% of C_2	% of C_3
Type A		at most 10%	
Type B	at least 30 %	at most 5%	at most 10%

   The selling price is 0.1 Euro/g for tablets of type A and 0.05 Euro/g for tablets of type B.

   Write the LP problem for the computation of the optimal quantity of chemicals that maximizes the profits under the assumption that all produced tablets will be sold.

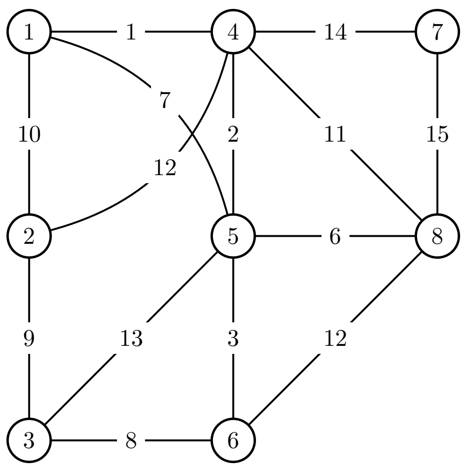
2. Consider the LP problem

$$
\begin{array}{rl}
\max_{x_1,x_2} & -6x_1 - x_2 \\
-x_1 + x_2 & \leq 2 \\
-3x_1 + x_2 & \leq -4 \\
x_2 & \geq 0
\end{array}
$$

   2.1 Write the dual problem and apply to it phase 1 of the simplex algorithm.

   2.2 Knowing that the multipliers obtained at the end of phase 1 are optimal for the dual problem, compute the optimal solution to the primal problem.

3. Compute a shortest spanning tree of the undirected network in the figure below.



4. A project is composed by the activities $A_i, i = 1, \ldots, 7$ that verify the immediate precedence relations

$$A_1 < A_2 \qquad A_1 < A_3 \qquad A_1 < A_4 \qquad A_2 < A_7$$
$$A_3 < A_5 \qquad A_4 < A_6 \qquad A_5 < A_6$$

The durations d_i of the activities are beta-distributed, mutually independent random variables whose mean and variance are given in the following table

	A_1	A_2	A_3	A_4	A_5	A_6	A_7
mean	1	3	1	2	3	4	4
variance	0.5	0.2	0.3	0.2	0.1	0.5	0.1

4.1 Perform the PERT analysis of the project.

4.2 Compute the 95% confidence interval of the minimal time needed for completing the project.

5. Determine whether the following statements are true or false. Scores: correct answer = 1, mistake = -0.5, no answer = 0.

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(a) Let $G = (V, E, k)$ be a flow network, where $k(e)$ is the capacity of the edge $e \in E$. Let ϕ_1 be the flow value computed at the end of an iteration of the Ford-Fulkerson algorithm and let ϕ_2 be the flow value computed at the end of the next iteration. Then $\phi_2 - \phi_1 \geq 0$.

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(b) A decision problem P is NP-hard if $\forall P_1 \in NP \; P_1 \propto P$.

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(c) Let $G = (V, E, c)$ be a directed network with strictly positive weights and assume to run Dijkstra's algorithm for computing shortest paths from a vertex $v_1 \in V$ to all other vertices. If during an iteration the vertex $\bar{v}$ becomes permanent, then at the beginning of the next iteration the label $l(\bar{v})$ is the cost of a shortest path from v_1 to $\bar{v}$

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(d) The union of two polytopes is a polytope.

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Nonlinear Systems

6. Consider the system

$$\dot{x}_1 = -3(x_1^3 + x_2^3)$$
$$\dot{x}_2 = x_1$$

6.1 Study the stability properties of the origin using the indirect Lyapunov method.

6.2 Study the stability properties of the origin using the candidate Lyapunov function $V(x) = x_1^2 + \alpha x_2^\beta$ where $\alpha, \beta \in \mathbb{R}$.

7. Consider the system

$$\dot{x}_1 = x_1^3 - x_2$$
$$\dot{x}_2 = x_2 + u$$

Verify the assumptions of the backstepping procedure and use it for designing a controller such that the origin of the closed-loop system is asymptotically stable. Compute a Lyapunov function certifying this property.