Industrial Automation - Advanced Automation and Control

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25 January 2011

Industrial Automation

1. A company wants to invest 300000 Euros using the financial products listed in the following table:

Financial product	Market	Return %
T_1	Food options - Germania	4.1
T_2	Food options - Cina	7.2
T_3	Food options - Francia	2.2
T_4	Pharmaceutical options - France	3.2
T_5	Pharmaceutical options - Cina	5.2
T_6	Italian bonds	1.2

For avoiding high risk exposure, the following rules must be followed:

- at most 100000 Euros can be invested in pharmaceutical options
- at least 100000 Euro must be invested in options in the food market
- at least 40% of the money invested in european options must be invested in Chinese options
- at most 70% of the money invested in bonds must be invested in options

Write the optimization problem for computing the optimal portfolio.

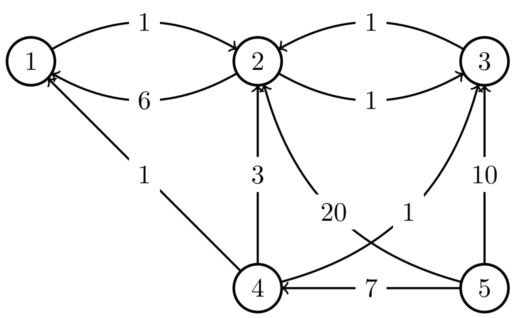
2. Consider the LP problem

$$\begin{aligned} \min_{x_1,x_2,x_3,x_4} \quad & 2x_1 + x_2 \\ -x_1 + x_2 + x_3 &= -1 \\ x_1 + x_2 + x_4 &= 3 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

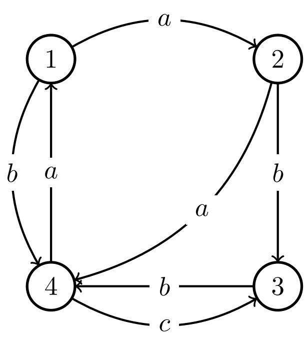
2.1 Verify that the basis associated to variables x_1 and x_3 is feasible and, starting from this basis, solve the problem using phase 2 of the simplex method.

2.2 Write the dual problem and compute optimal multipliers using complementary slackness conditions.

3. Consider the directed network in the picture below. Compute shortest paths from vertex 5 to all other vertices.



4. Consider the automaton in the figure below.



where $C = \{a, b, c\}$ is the set of control values and $S = \{1, 2, 3, 4\}$ is the set of state values. Let $g(x, u)$ be the intermediate cost specified in the following table

	a	b	c
1	1	0	-
2	2	2	-
3	-	3	-
4	1	-	1

and let g_2 be the terminal cost defined as

$$g_2(x) = \begin{cases} 0 & \text{se } x = 1 \\ 1 & \text{se } x = 2 \\ 2 & \text{se } x = 3 \\ 0 & \text{se } x = 4 \end{cases}$$

4.1 Solve the optimal control problem

$$J(x_0) = \min_{u_0, u_1} g_2(x_2) + \sum_{k=0}^{1} g(x_k, u_k)$$

using dynamic programming.

4.2 Compute an optimal control sequence for $x_0 = 3$ and compute the optimal cost value.

5. Determine whether the following statements are true or false. Score: correct answer = 1, mistake = −0.5, no answer = 0.

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(a) Let S_1 and S_2 be two decision problems where S_1 is NP-hard and S_2 is polynomial. If $S_1 \propto S_2$ then it holds $P = NP$.

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(b) Let P be a convex programming problem. If constraints of P are qualified, then strong duality holds.

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(c) Let $G = (V, E, k)$ be a flow network ($k(e)$ is the capacity of the edge $e \in E$) and let x be a feasible flow. If the set of edges of the residual network corresponding do x coincides with E then the flow value is zero.

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(d) Let G be an undirected acyclic graph. Then G is a tree.

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Nonlinear Systems

6. Consider the system

$$\dot{x}_1 = x_2 - 1$$
$$\dot{x}_2 = -x_1^2 x_2^3 + x_1$$

6.1 Compute equilibrium states, classify them and, if possible, sketch the qualitative portrait of state trajectories in their neighborhood.

6.2 Is it possible to have a closed orbit embracing only a single equilibrium state or all of them ?

7. Consider the first-order system

$$\dot{x}_1 = x_1^3 + x_1 u$$
$$y = x_1$$

Design, if possible, a regulator guaranteeing tracking of a constant setpoint $y^o \in [1, 2]$.