Industrial Automation - Advanced Automation and Control

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Industrial Automation

1. A food company blends four ingredients (C_1, C_2, C_3, C_4) for obtaining sport energy tablets of type A, B and C. The maximal number of available kilograms of each ingredient and the associated prices are given in the following table.

	C_1	C_2	C_3	C_4
Availability (Kg)	100	200	250	400
Price (Euro / Kg)	0.5	0.1	0.3	0.5

The produced tablets must fulfill the following constraints on the percentages of each ingredient.

	% of C_1	% of C_2	% of C_3	% of C_4
Type A	no more than 10%	exactly 10%		
Type B	at least 30 %		no more than 10%	
Type C		exactly 30 %		at least 30 %

Selling prices are 20 Euro/kg for tablets A, 30 Euro/kg for tablets B and 35 Euro/kg for tablets C.

Write the optimization problem that allows one to find the optimal quantities of ingredients for maximizing the profits due to the selling of the tablets.

2. Consider the LP problem

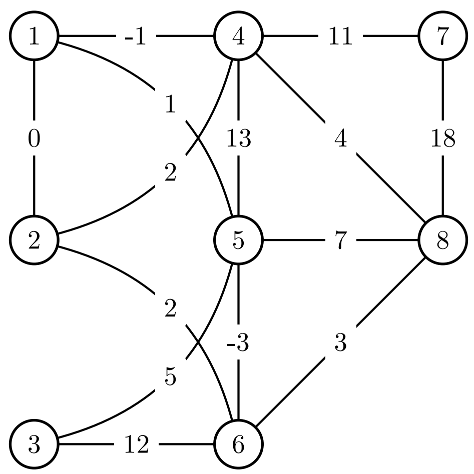
$$\begin{aligned} \min_{x_1,x_2} \quad & -\tfrac{1}{3}x_1 + \tfrac{1}{3}x_2 \\ -x_1 - x_2 &\leq 5 \\ -2x_1 + x_2 &\geq -2 \\ x_1 &\leq 0 \\ x_2 &\leq 0 \end{aligned}$$

2.1 Draw the feasible region and compute an optimal solution in a graphical way.

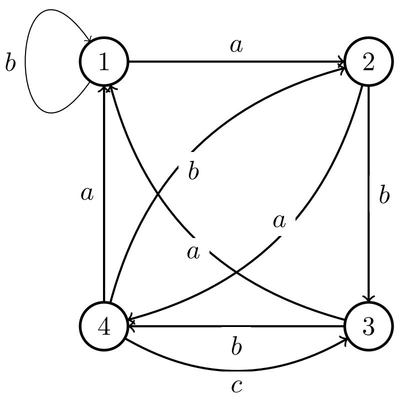
2.2 Write the LP problem in standard form with $b \geq 0$. Verify that the basis associated with slack and excess variables is feasible. Starting from this basis, execute phase 2 of the simplex algorithm in the tableau form.

2.3 Write the dual problem.

3. Compute a shortest spanning tree of the undirected network below. Is it unique ?



4. Consider the automaton in the figure below.



where $C = \{a, b, c\}$ is the set of control values and $S = \{1, 2, 3, 4\}$ is the set of state values. Let the intermediate cost $g(x, u)$ and the terminal cost $g_2(x)$ be given by

$g(x,u)$	a	b	c
1	2	6	-
2	3	5	-
3	1	8	-
4	4	2	3

$$g_2(x) = \begin{cases} 1 & \text{if } x = 1 \\ 4 & \text{if } x = 2 \\ 4 & \text{if } x = 3 \\ 1 & \text{if } x = 4 \end{cases}$$

4.1 Solve the optimal control problem

$$J(x_0) = \min_{u_0, u_1} g_2(x_2) + \sum_{k=0}^{1} g(x_k, u_k)$$

using dynamic programming.

4.2 Compute an optimal control sequence for $x_0 = 4$ and compute the optimal cost value.

5. Determine whether the following statements are true or false. Scores: correct answer $= 1$, wrong answer $= -0.5$, no answer $= 0$.

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(a) Assume two convex optimization problems have identical dual problems. If x^* is a primal optimizer of the first problem and $\tilde{x}$ is a primal optimizer of the second problem, then $x^* = \tilde{x}$.

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(b) If P_1 and P_2 are NP-complete decision problems, then one has that, simultaneously, $P_1 \propto P_2$ and $P_2 \propto P_1$.

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(c) Let $G = (V, E, c)$ be a directed network with strictly positive weights and assume one uses Dijkstra's algorithm for computing shortest paths from $v_1 \in V$ to all other nodes. Assume v is a permanent node at the end of an iteration.Then, $l(\bar{v})$ is the cost of a shortest path from v_1 to $\bar{v}$.

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(d) Let $G = (V, E, k)$ be a flow network (where $k(e)$ is the capacity of the edge e) and let x be a feasible flow. If $x(e) < k(e)$, $\forall e \in E$, then no residual capacity is zero in the residual network corresponding to x.

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6. Consider the system

$$\dot{x}_1 = -\sin x_1 + x_2^3$$
$$\dot{x}_2 = -x_1^3$$

6.1 Using the indirect Lyapunov method, study the stability of the origin and, if possible, classify it.

6.2 Study the stability of the origin using the candidate Lyapunov function $V(x) = x_1^4 + \alpha x_2^4$ where $\alpha \in \mathbb{R}$.

7. Consider the system

$$\dot{\theta}_1 = \theta_1 + \theta_2$$
$$\dot{\theta}_2 = 3\theta_1^2\theta_2 + \theta_1 + \tau$$

where $\tau(t)$ is an input. Design a state-feedback controller such that the state $\bar{\theta} = [1,\ -1]^T$ is a closed-loop asymptotically stable equilibrium.