Industrial Automation - Advanced Automation and Control

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Industrial Automation

1. A company produces 4 types of car engines (E_i, $i = 1, \dots 4$) that must go through 2 production lines (l_1 and l_2). In order to process a single engine, in each production line the men-hours specified in the following table are required

	l_1	l_2
E_1	3	4
E_2	7	9
E_3	10	8
E_4	2	1

Knowing that

- at least 100 engines of type E_1, and no more than 50 engines of type E_4 must be produced;
- man-hours fall within the three categories T_i, $i = 1, \dots, 3$ according to the versatility of workers, as specified in the following table

Category	Line	Maximal availability (men-hours)
T_1	l_1	320
T_2	l_2	400
T_3	$l_1,\ l_2$	000

- profits per unit are 300 Euros for engine E_1 , 350 Euros for engine E_2 , 400 Euros for engine E_3 , 450 Euros for engine E_4

and assuming all products will be sold, write the LP problem that allows one to determine the production and resource allocation plan maximizing the profits.

2. Consider the LP problem

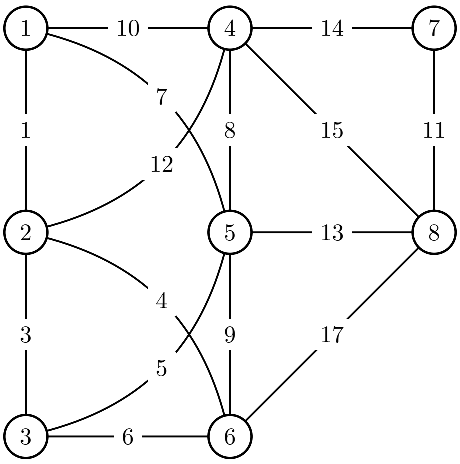
$$
\begin{array}{rl}
\min_{x_1,x_2} & -\frac{2}{5}x_1 + x_2 \\
\frac{5}{2}x_1 - x_2 & \leq -10 \\
-x_1 + x_2 & \geq 7 \\
x_1 & \leq 0 \\
x_2 & \geq 0
\end{array}
$$

2.1 Draw the feasible region and compute the optimal solution in a graphical way.

2.2 Run phase 1 of the simplex method in the tableau form.

2.3 Write the dual problem and find optimal multipliers using complementary slackness conditions.

3. Compute a shortest spanning tree of the undirected network in the figure below.



4. A project is composed by the activities $A_i, i = 1, \ldots, 8$ that verify the immediate precedence relations

$$A_1 < A_3 \qquad A_1 < A_6 \qquad A_2 < A_4 \qquad A_2 < A_8$$
$$A_3 < A_5 \qquad A_4 < A_5 \qquad A_5 < A_7 \qquad A_6 < A_7$$

The durations of the activities are given in the following table

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
durations	3	1	2	6	2	5	4	7

Provide an AOA representation of the project. Compute also early and late start times of each activity and a critical path.

5. Determine whether the following statements are true or false. Scores: correct answer = 1, wrong answer = −0.5, no answer = 0.

T F

(a) Let $G = (V, E, k)$ be a flow network, where $k(e)$ is the capacity of the edge $e \in E$. Let x be a feasible and maximal flow. Then, in the residual network there is no path from the source node to the destination node.

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(b) Let P_1 and P_2 be decision problems. If P_1 is NP, P_2 is NP-complete and $P_2 \propto P_1$, then P_1 is NP-complete.

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(c) Assume that the optimization problem $\{\min_{x \in \mathbb{R}^n} f(x) : g(x) \leq 0, h(x) = 0\}$ is convex, f, g, h are twice continuously differentiable and x^*, λ^*, μ^* verify the KKT conditions. Then, x^* is an optimal solution.

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(d) An undirected connected graph T such that the deletion of any arc makes T acyclic is a tree.

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Nonlinear Systems

6. Consider the system

$$\dot{x}_1 = x_2^2 - 2x_2 + 1 - x_1^3$$
$$\dot{x}_2 = -x_1 x_2 + x_1$$

6.1 Compute equilibrium states and classify them, if possible.

6.2 Study the stability of equilibrium states using the Lyapunov function $V(x) = \tilde{x}_1^2 + \tilde{x}_2^2$, where $\tilde{x}_1, \tilde{x}_2$ are suitable coordinates.

7. Consider the system

$$\dot{x}_1 = 4x_1 + e^{x_1} x_2$$
$$\dot{x}_2 = x_2^2 x_1 - (x_2^2 + 1)u$$

Verify the assumptions of the backstepping procedure and use it for designing a controller such that the origin of the closed-loop system is asymptotically stable. Compute a Lyapunov function certifying this property.