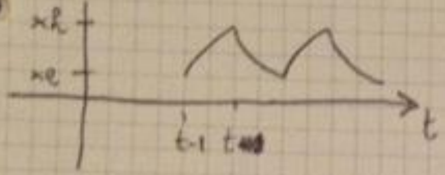
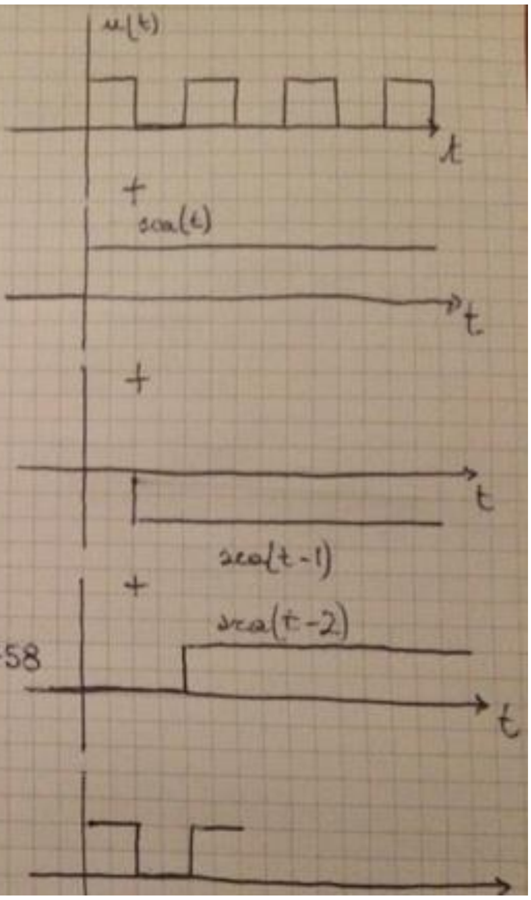
$\dot{x}(t) = -0.1\,x(t) + 0.5u(t)$



x_e?

x_h?

esiste?



Ipotizziamo esista

$x_h = e^{-0.1(t-t+1)} \cdot x_e + \int_{t-1}^{t} e^{-0.1(t-\tau)}\, 0.5\, d\tau$

$x_h = e^{-0.1} x_e + 5e^{-0.1t}\left[e^{0.1t} - e^{0.1t-0.1} \right]$

$x_h = e^{-0.1} x_e + 5\left[1 - e^{-0.1} \right] = e^{-0.1} x_e + 0.4758$

$x_e = e^{-0.1} x_h \rightarrow x_h = e^{0.1} x_e$

$e^{0.1} x_e = e^{-0.1} x_e + 0.4758$

$x_e\left(e^{0.1} - e^{-0.1} \right) = 0.4758$

$x_e = 0.4758 / \left(e^{0.1} - e^{-0.1} \right) = 2.3751$

$x_h = e^{-0.1}\, 2.3751 + 0.4758 = 2.6249$ ← feasible

$u(t) = sca(t) - sca(t-1) + sca(t-2) \cdots$

Quando ci arrivo?

Principio sovrapposizione effetti

$x^1(0) = 0 \qquad u^1(t) = sca(t)$

$x^2(0) = 0 \qquad u^2(t) = -sca(t-1)$

$\vdots$

$\alpha^i = 1 \qquad x(t) = \sum \alpha_i x^i(t)$

$x(0) = \sum \alpha_i x^i(0)$

$u(t) = \sum \alpha_i u^i(t)$

Visto che $x^i(0) = 0$

$x^1(t) = 5\left[1 - e^{-0.1t} \right]$

$x^2(t) = +5\left[-1 + e^{-0.1(t-1)} \right]$

$x^3(t) = 5\left[1 - e^{-0.1(t-2)} \right]$

$\vdots$

$x(t) = 5 \sum_{i=0}^{t} (-1)^i \left(1 - e^{-0.1(t-i)} \right)$

$x(t) = 5 \sum_{i=0}^{t} (-1)^i - 5 \sum_{i=0}^{t} e^{-0.1t} \cdot e^{0.1i} \cdot (-1)^i =$

$= 5\left[\sum_{i=0}^{t} (-1)^i - e^{-0.1t} \sum_{i=0}^{t} \left(-e^{0.1} \right)^i \right] = 5\left[\frac{1-(-1)^{t+1}}{2} - e^{-0.1t}\left(\frac{1-(-e^{0.1})^{t+1}}{1+e^{0.1}} \right) \right] =$

$$5\left[\frac{1-(1)^{t+1}}{2} - e^{-0.1t}\left(\frac{1-(-e^{0.1})^{t+1}}{1+e^{0.1}}\right)\right]$$

supponiamo $t+1$ pari

$$5\left[0 - e^{-0.1t}\left[\frac{1-e^{0.1t+0.1}}{1+e^{0.1}}\right]\right]$$

$$5\left[\frac{-e^{-0.1t}+e^{0.1}}{1+e^{0.1}}\right] \xrightarrow{t\to\infty} \quad (t \text{ dispari})$$

$$e^{-0.1t} \to 0 \to 5\cdot\left(\frac{e^{0.1}}{1+e^{0.1}}\right) = 2.6249$$

ci arrivo asintoticamente