Calcolo dose e intervallo di somministrazione delle pastiglie

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Formula per il calcolo della matrice esponenziale

$$e^{At} = \sum_{i=1}^{n} e^{\lambda_i t} \mathcal{F}(\lambda_i), \quad \mathcal{F}(\lambda_i) = \prod_{j=0, j\neq i}^{n} \frac{A - \lambda_j I}{\lambda_i - \lambda_j}$$

$$A = \begin{bmatrix} -1 & 0 \\ 1 & -0.5 \end{bmatrix}, B = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D = 0$$

$$e^{At} = -2e^{-t} \begin{bmatrix} -0.5 & 0 \\ 1 & 0 \end{bmatrix} + 2e^{-0.5t} \begin{bmatrix} 0 & 0 \\ 1 & 0.5 \end{bmatrix} = \begin{bmatrix} e^{-t} & 0 \\ -2e^{-t} + 2e^{-0.5t} & e^{-0.5t} \end{bmatrix}$$

Calcolo il movimento di x_1

$$\begin{aligned} x_1(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} e^{At} \left(\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} + \begin{bmatrix} 0.5\bar{u} \\ 0 \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 \\ -2e^{-t} + 2e^{-0.5t} & e^{-0.5t} \end{bmatrix} \begin{bmatrix} x_1(0) + 0.5\bar{u} \\ y_m \end{bmatrix} = \\ &= \begin{bmatrix} e^{-t} & 0 \end{bmatrix} \begin{bmatrix} x_1(0) + 0.5\bar{u} \\ y_m \end{bmatrix} = e^{-t} \left(x_1(0) + 0.5\bar{u} \right) \end{aligned}$$

Definisco

$$\alpha := x_1(0) + 0.5\bar{u}$$

Impongo che $x_1(\Delta t) = x_1(0)$

$$\begin{aligned} x_1(\Delta t) &= x_1(0) = e^{-\Delta t} \left(x_1(0) + 0.5\bar{u} \right) \\ e^{-\Delta t} 0.5\bar{u} &= (\alpha - 0.5\bar{u}) (1 - e^{-\Delta t}) \\ \bar{u} \left(e^{-\Delta t} 0.5 + 0.5(1 - e^{-\Delta t}) \right) &= \alpha (1 - e^{-\Delta t}) \\ \bar{u} &= 2\alpha (1 - e^{-\Delta t}) \end{aligned}$$

Calcolo il movimento dell'uscita partendo con $x_2 = y_m$ (valore minimo della concentrazione 0.45)

$$\begin{aligned}
y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} e^{At} \begin{bmatrix} \alpha \\ y_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 \\ -2e^{-t} + 2e^{-0.5t} & e^{-0.5t} \end{bmatrix} \begin{bmatrix} \alpha \\ y_m \end{bmatrix} = \\
&= \begin{bmatrix} -2e^{-t} + 2e^{-0.5t} & e^{-0.5t} \end{bmatrix} \begin{bmatrix} \alpha \\ y_m \end{bmatrix} = -2\alpha e^{-t} + 2\alpha e^{-0.5t} + e^{-0.5t} y_m \\
&= -2\alpha e^{-t} + (2\alpha + y_m) e^{-0.5t}
\end{aligned}$$

Calcolo il valore massimo

$$\dot{y}(t) = 2\alpha e^{-t} - (\alpha + 0.5 y_m) e^{-0.5t}$$

Definisco

$$p_M := e^{-0.5 t_M}$$

Impongo la derivata a zero

$$\dot{y}(t_M) = 2\alpha p_M^2 - (\alpha + 0.5 y_m) p_M = 0$$

Scelgo la radice non nulla

$$p_M = \frac{(\alpha + 0.5 y_m)}{2\alpha} = e^{-0.5 t_M}$$

Impongo che $y(t_M) = y_M = 0.55$

$$\begin{gathered}
y(t_M) = -2\alpha p_M^2 + (2\alpha + y_m) p_M = y_M \\
(\alpha + 0.5 y_m)^2 - (2\alpha + y_m)(\alpha + 0.5 y_m) + y_M 2\alpha = 0 \\
\alpha^2 + 0.25 y_m^2 + \alpha y_m - 2\alpha^2 - \alpha(y_m + y_m) - 0.5 y_m^2 + y_M 2\alpha = 0 \\
\alpha^2 + \alpha(y_m - 2 y_M) + 0.25 y_m^2 = 0
\end{gathered}$$

Scelgo la radice più grande altrimenti poi t_M sarebbe negativo

$$\alpha = 0.5595$$

$$t_M = -2 \log(p_M) = 0.7103$$

Impongo che $y(\Delta t) = y_m$

$$y(\Delta t) = -2\alpha e^{-\Delta t} + (2\alpha + y_m) e^{-0.5\Delta t} = y_m$$

Definisco

$$\begin{gathered}
p_\Delta = e^{-0.5\Delta t} \\
-2\alpha p_\Delta^2 + (2\alpha + y_m) p_\Delta = y_m \\
2\alpha p_\Delta^2 - (2\alpha + y_m) p_\Delta + y_m = 0
\end{gathered}$$

Scelgo la radice diversa da zero (altrimenti Δt sarebbe uguale a zero)

$$p_\Delta = 2.4868$$

$$\Delta t = -2 \log(p_\Delta) = 1.8220$$

La dose risulta quindi

$$\bar{u} = 2\alpha(1 - e^{-\Delta t}) = 0.9381$$