Università degli Studi di Pavia
Dipartimento di Ingegneria Industriale e dell'Informazione

Corso di Identificazione dei Modelli e Analisi dei Dati

Random Variables (part 2)

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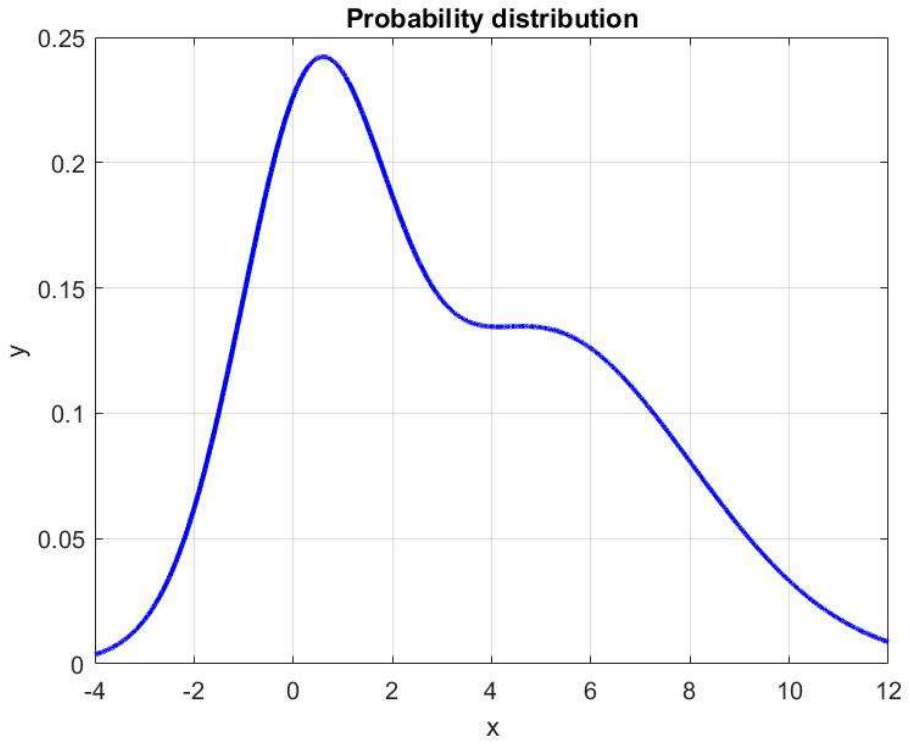
Outline

1. Theoretical parameters of a random variable
2. Sample parameters of a random variable
3. Sample mean and sample median

Parameters of a random variable

- Mode
- Mean
- Median
- Quantiles

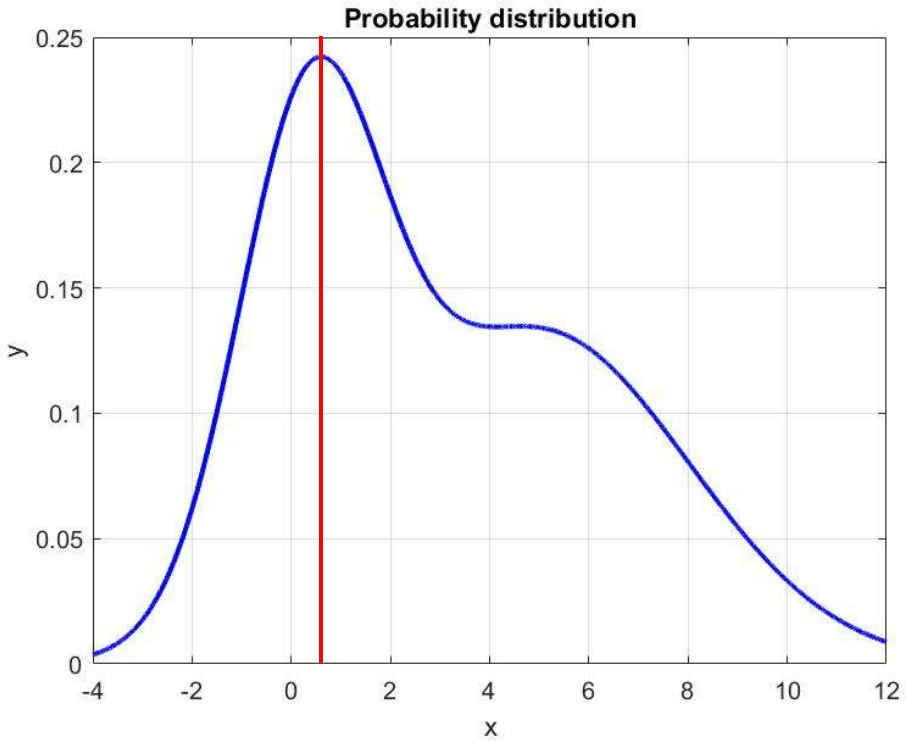


Parameters of a random variable

- **Mode**

$$\underset{x}{\operatorname{argmax}}\left(f_X(x)\right)$$

Cons: 1) It may not be unique.
2) It may not be significant.

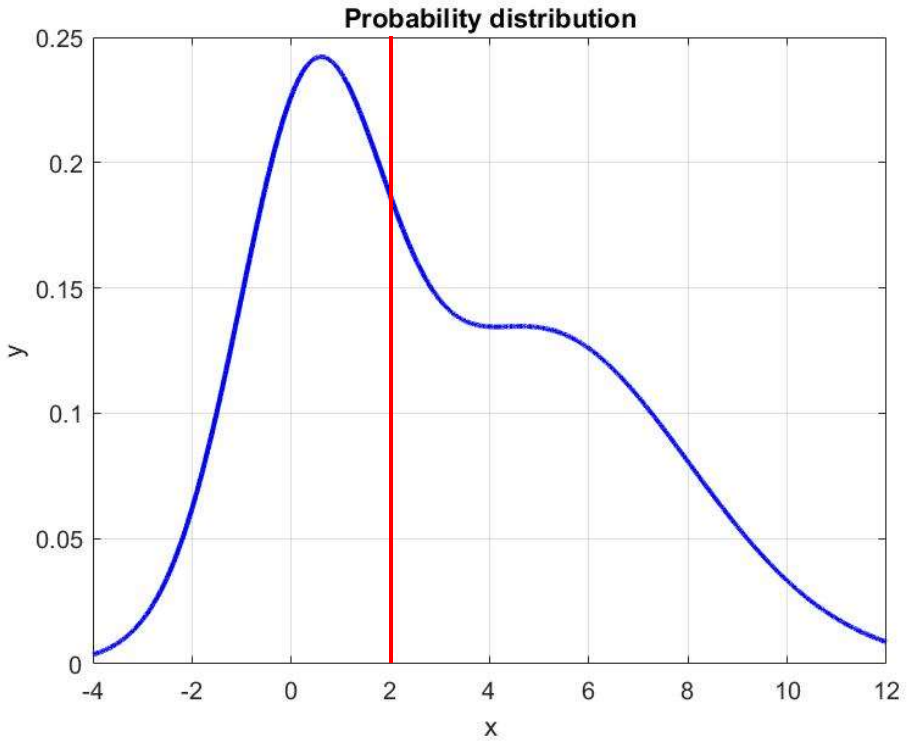


Parameters of a random variable

- **Mean**

$$E(X) = \int_{-\infty}^{\infty} x f_X(x) dx$$

It is the centroid of the distribution.

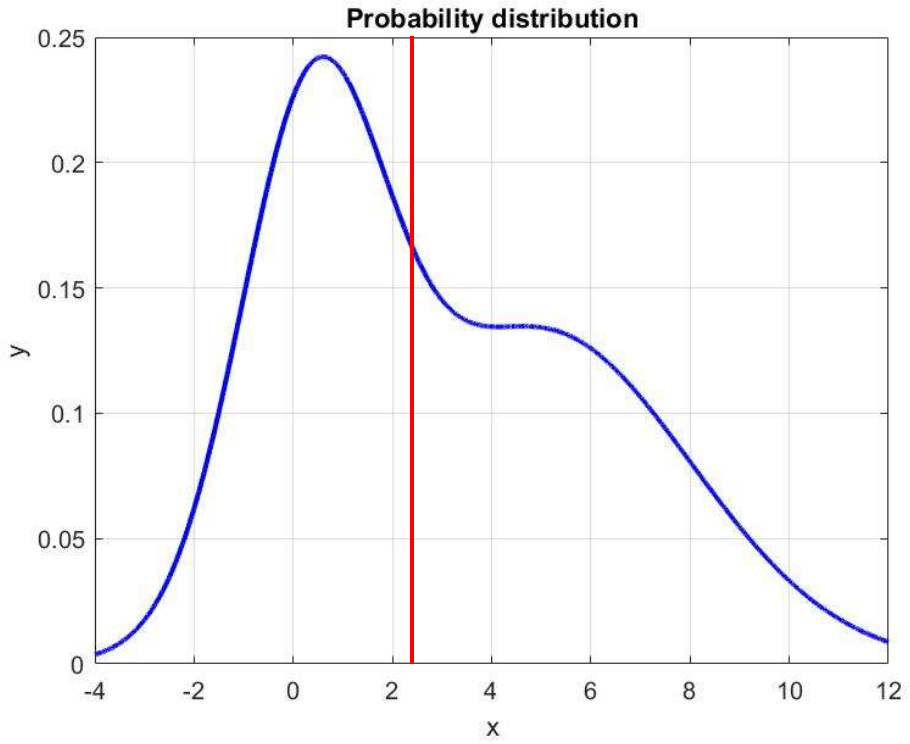


Parameters of a random variable

- **Median**

$$F_X^{-1}(0.5)$$

Cons: It may not be unique.



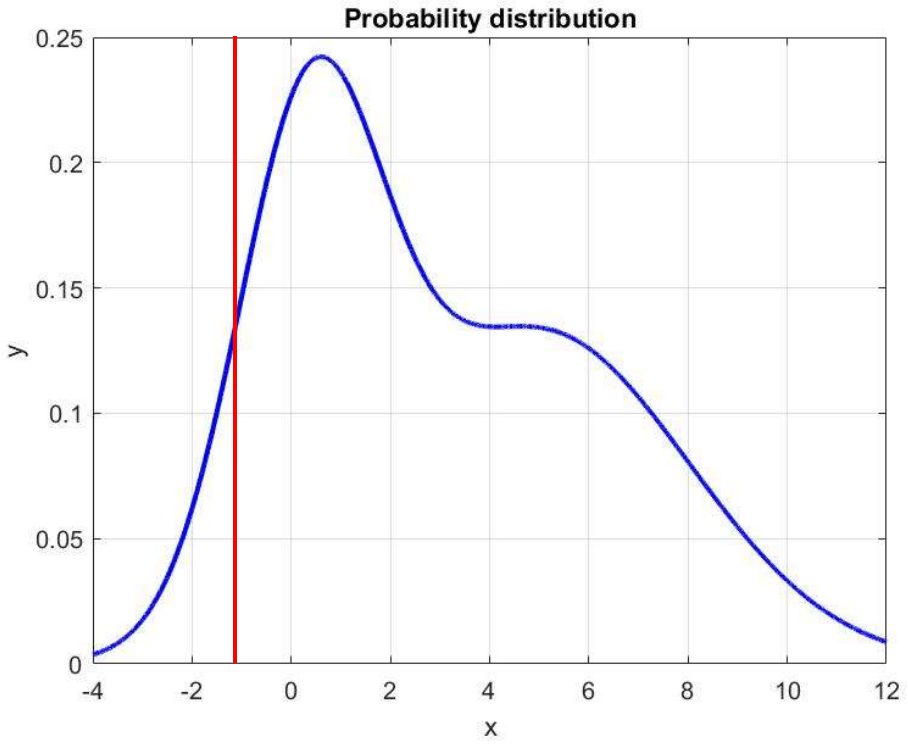
Parameters of a random variable

- **Quantiles**

e.g.: the 0.25-quantile
(i.e. the first quartile)

$$F_X^{-1}(0.25)$$

The median is the quantile $x_{0.5}$



Calculate quantiles with MATLAB

$$F_X(x_\alpha) = \alpha \implies x_\alpha = F_X^{-1}(\alpha)$$

Cumulative Distribution Function

Inverse Cumulative Distribution Function

doc icdf

Exercise 1

1. Create a Normal distribution object with m = 1 and sigma = 2.

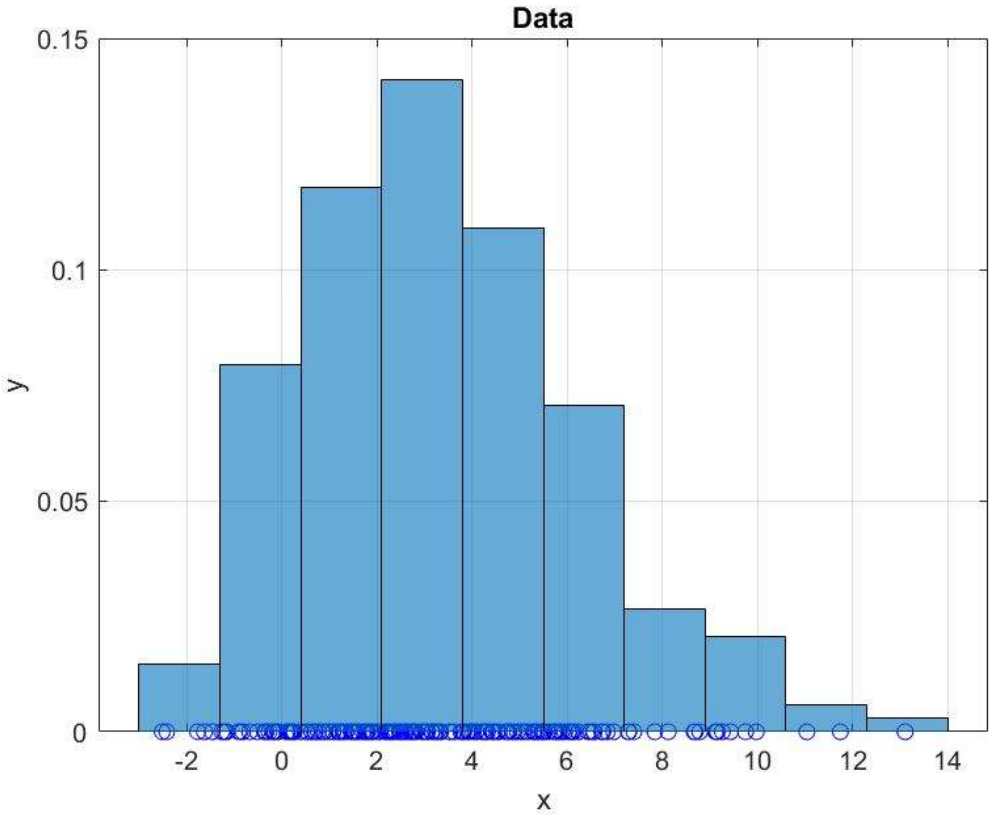
2. Calculate the theoretical median, first quartile and third quartile using icdf command.

3. Plot the theoretical distribution (use linspace to create a grid of x-values and pdf to obtain the y-values).

4. Plot, in the same graph, 4 vertical asymptote in correspondence of the theoretical mean, median, first quartile and third quartile.

Estimate the parameters of a distribution from the data

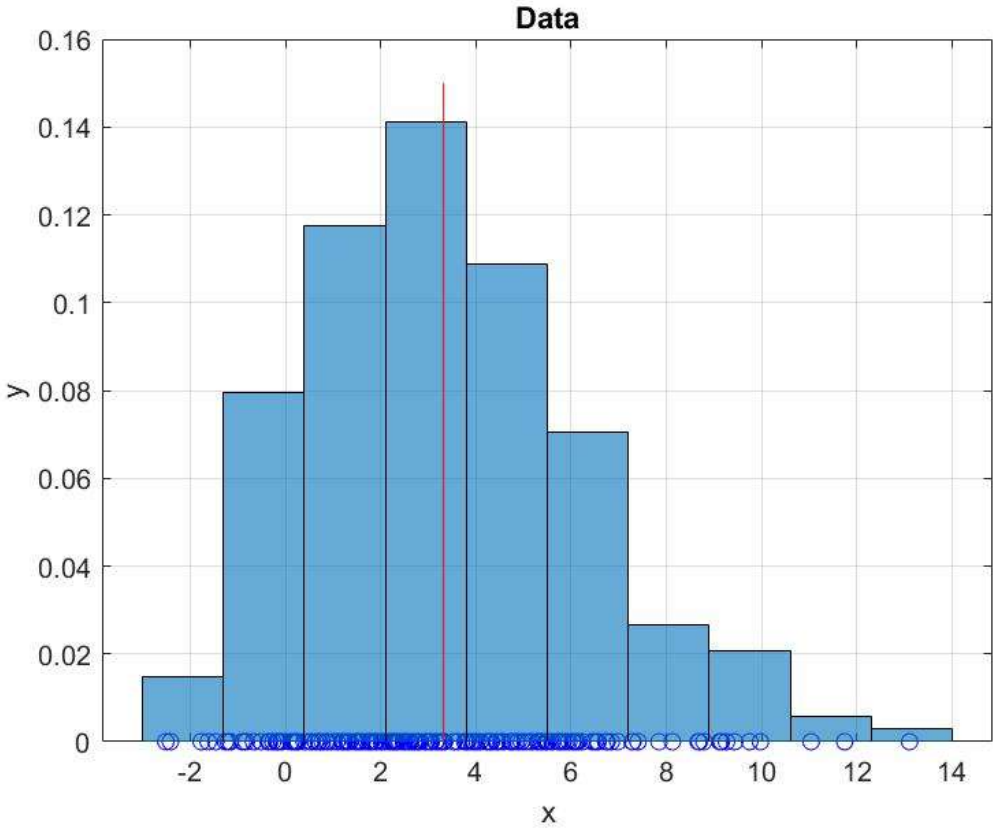
- Sample mean
- Sample median
- Sample quantiles



Estimate the parameters of a distribution from the data

- **Sample mean**

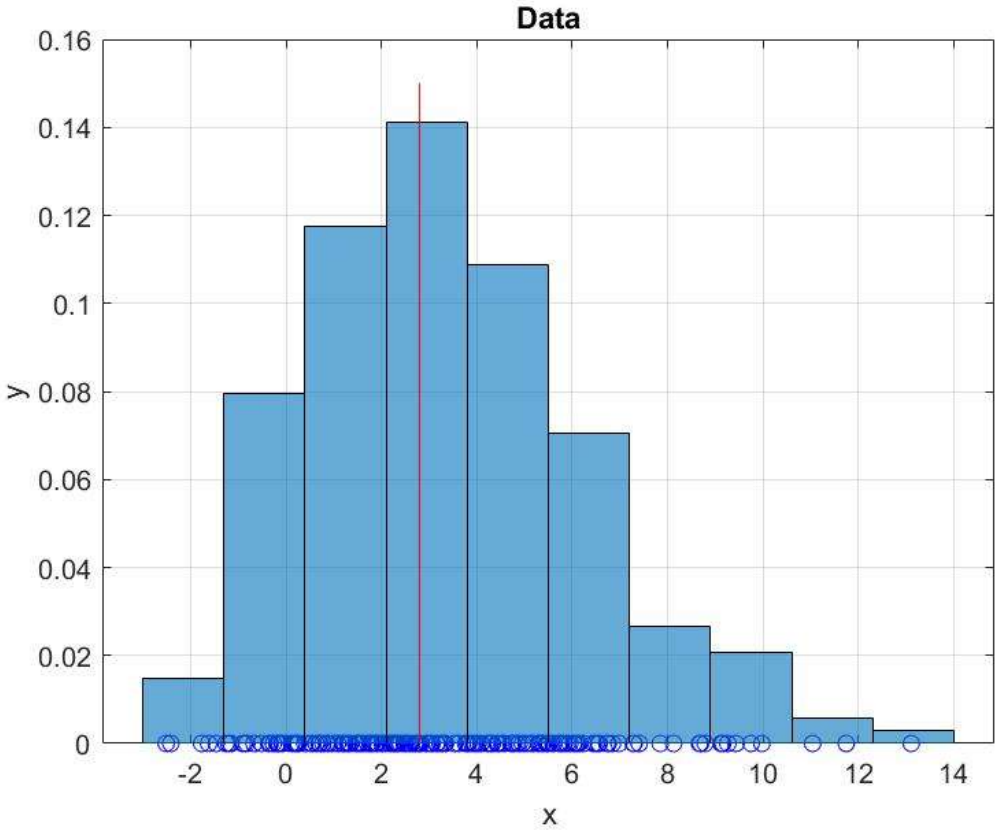
$$\frac{1}{N}\sum_{i=1}^{N} x_i$$



Estimate the parameters of a distribution from the data

- **Sample median**

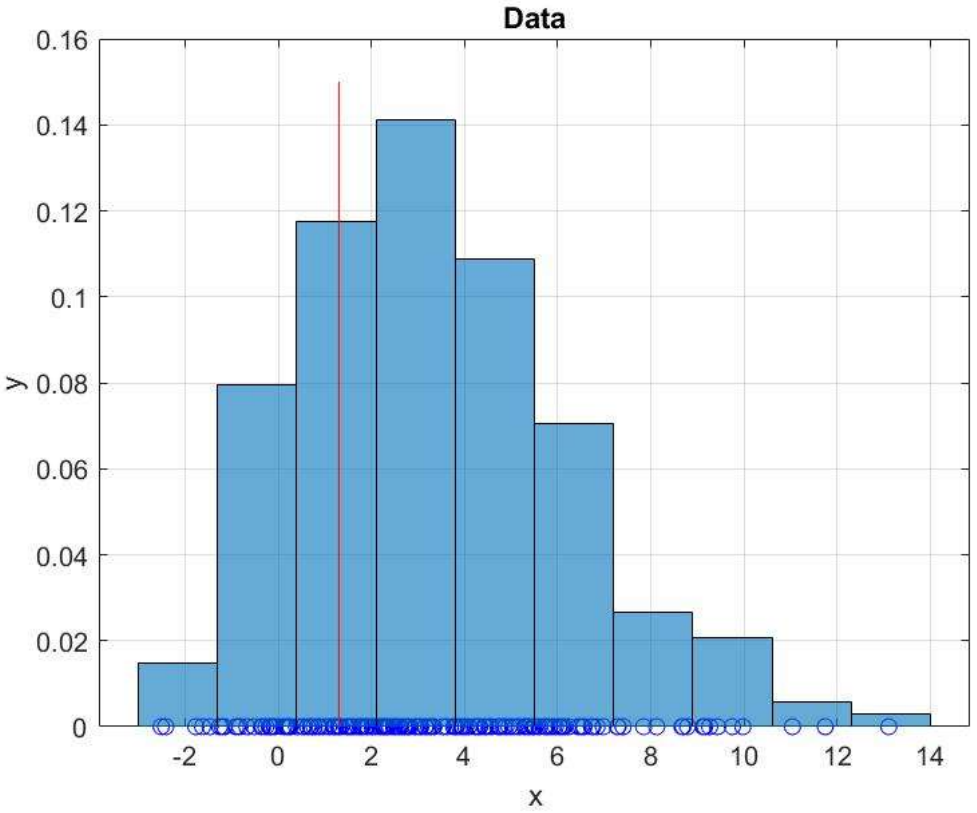
The ‘’middle’’ value



Estimate the parameters of a distribution from the data

- **Sample quantiles**

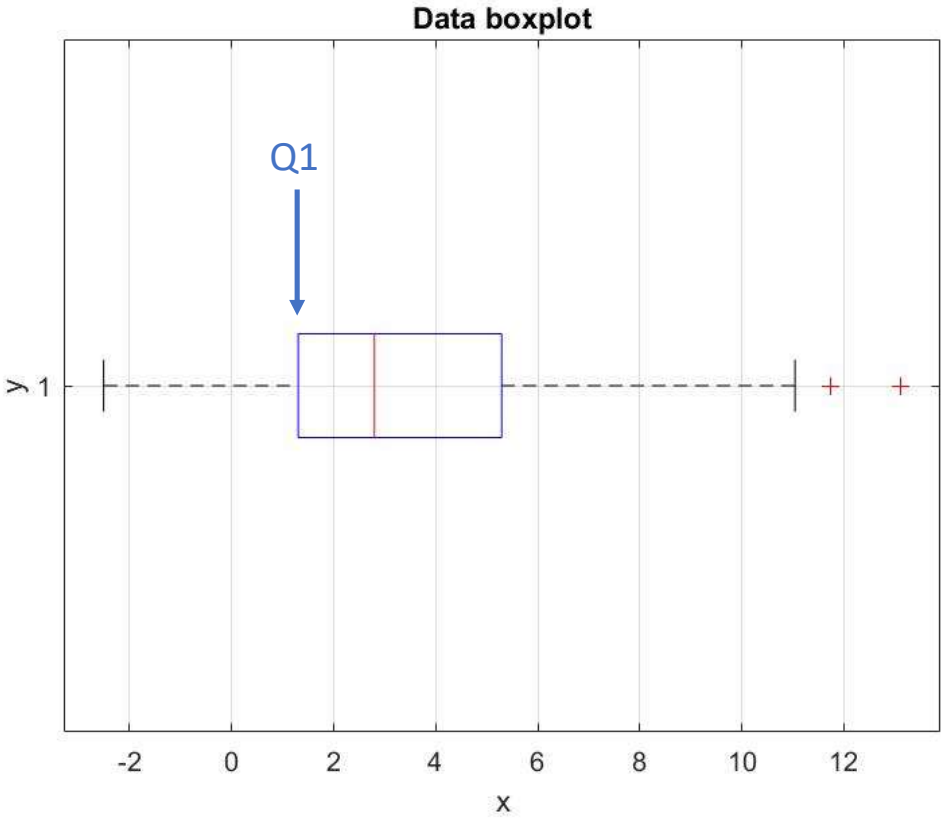
Sorting-based algorithm (see documentation for quantile)



Estimate the parameters of a distribution from the data

- **Sample quantiles**

Sorting-based algorithm (see documentation for quantile)



Calculate sample parameters with MATLAB

Sample mean: → doc mean

Sample median: → doc median

Sample quantile: → doc quantile

Exercise 2

1. Create a Log-normal distribution object with m = 1 and sigma = 0.5.
2. Calculate the theoretical mean, median, first quartile and third quartile.
3. Plot the theoretical distribution (use linspace to create a grid of x-values and pdf to obtain the y-values)→ for the grid use: x_grid = linspace(m - 4*sigma, m + 16*sigma, 10000).
4. Plot, in the same graph, the theoretical mean as a vertical asymptote.
5. Simulate 1000 random values from the theoretical distribution and compute the sample mean.
6. Plot, in the same graph, the sample mean as a vertical asymptote.
7. Repeat the exercise considering the median instead of the mean.

Sample mean and sample median

The sample mean accuracy increases with the number of samples (without outliers).

The sample median is robust against outliers. If you throw away the largest and smallest values in a data set then the **median** does not change but **the sample mean** does.

Exercise 3

1. Repeat the Exercise 2 with a bigger dataset and then with a smaller dataset.

2. Add an outlier to the simulated data (use the square brackets to concatenate a new value to the data vector generated with random) and recompute and plot the sample mean.

3. Repeat the exercise considering the median.

Reference Documentation:

- https://it.mathworks.com/

- http://sisdin.unipv.it/labsisdin/teaching/courses/imadlt/esercitazioni