Università degli Studi di Pavia
Dipartimento di Ingegneria Industriale e dell'Informazione

Corso di Identificazione dei Modelli e Analisi dei Dati

Multivariate Random Variables

Prof. Giuseppe De Nicolao, Federica Acerbi, Alessandro Incremona

Multivariate normal distributions

- **probability density function (pdf)**

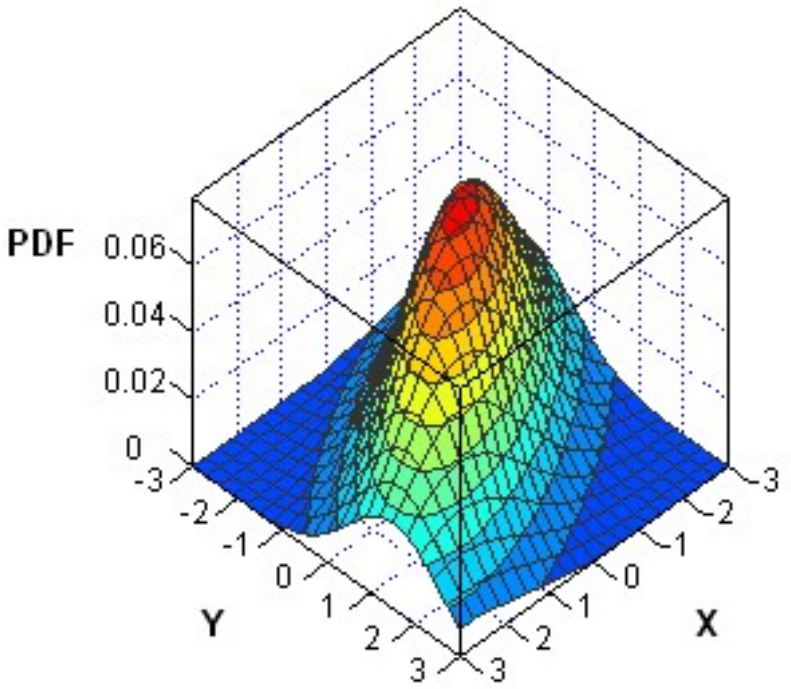
  >> y = mvnpdf(X,MU,SIGMA)

- **cumulative distribution function (cdf)**

  >> c = mvncdf(X,MU,SIGMA)

- **random numbers generation**

  >> r = mvnrnd(MU,SIGMA,n)



Exercise 1 (part 1)

1. Generate 10000 pairs of values (X,Y) from a bivariate normal distribution $\mathcal{N}(\mu, \Sigma)$,

   with $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 0.6 \\ 0.6 & 2 \end{bmatrix}$

Exercise 1 (part 1)

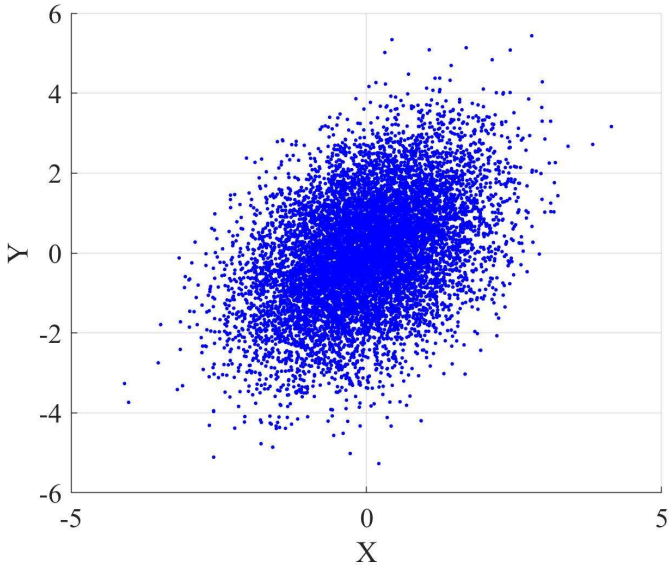
1. Generate 10000 pairs of values (X,Y) from a bivariate normal distribution $\mathcal{N}(\mu, \Sigma)$,

with $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 0.6 \\ 0.6 & 2 \end{bmatrix}$

Solution:

```
>> mean_vec = [0 0];
>> cov_mat = [1 0.6; 0.6 2];
>> rng(2)
>> data = mvnrnd(mean_vec, cov_mat, 10000);

>> figure(1)
>> scatter(data(:,1), data(:,2),'.b')
```



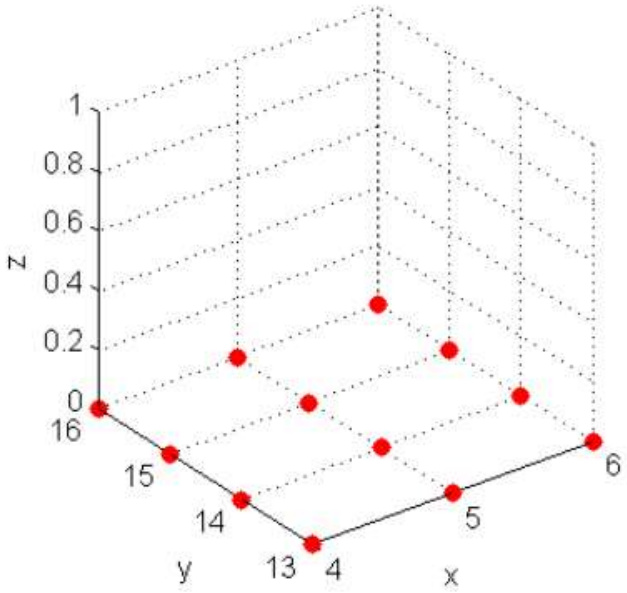
3-D Surface Plots

```
>> x = 4:6

     4     5     6

>> y = 13:16

    13    14    15    16
```

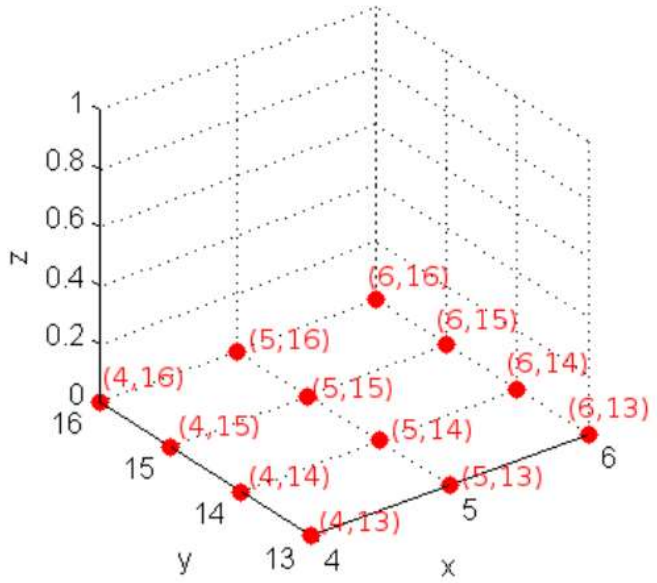


3-D Surface Plots

```
>> [X,Y] = meshgrid(x,y)
```

$$X = \begin{bmatrix} 4 & 5 & 6 \\ 4 & 5 & 6 \\ 4 & 5 & 6 \\ 4 & 5 & 6 \end{bmatrix} \qquad Y = \begin{bmatrix} 13 & 13 & 13 \\ 14 & 14 & 14 \\ 15 & 15 & 15 \\ 16 & 16 & 16 \end{bmatrix}$$

Notice that each x-value is paired with each y-value at some location within the arrays.



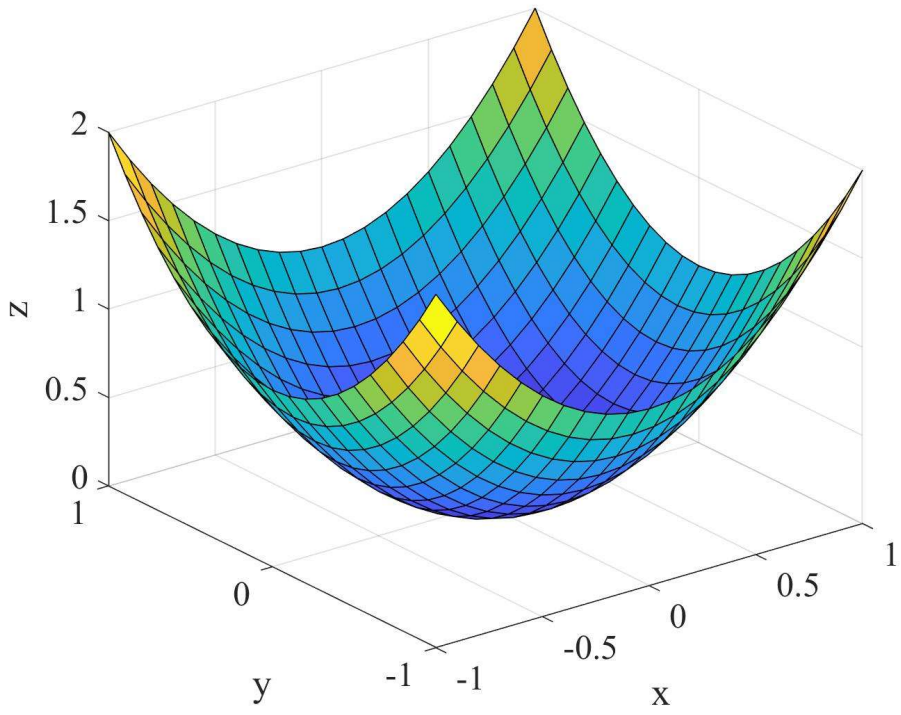
3-D Surface Plots

```
>> x = -1:0.1:1;
>> y = -1:0.1:1;

>> [X,Y] = meshgrid(x,y);

>> Z = X.^2 + Y.^2;
>> surf(X,Y,Z)

>> xlabel('x')
>> ylabel('y')
>> zlabel('z')
```

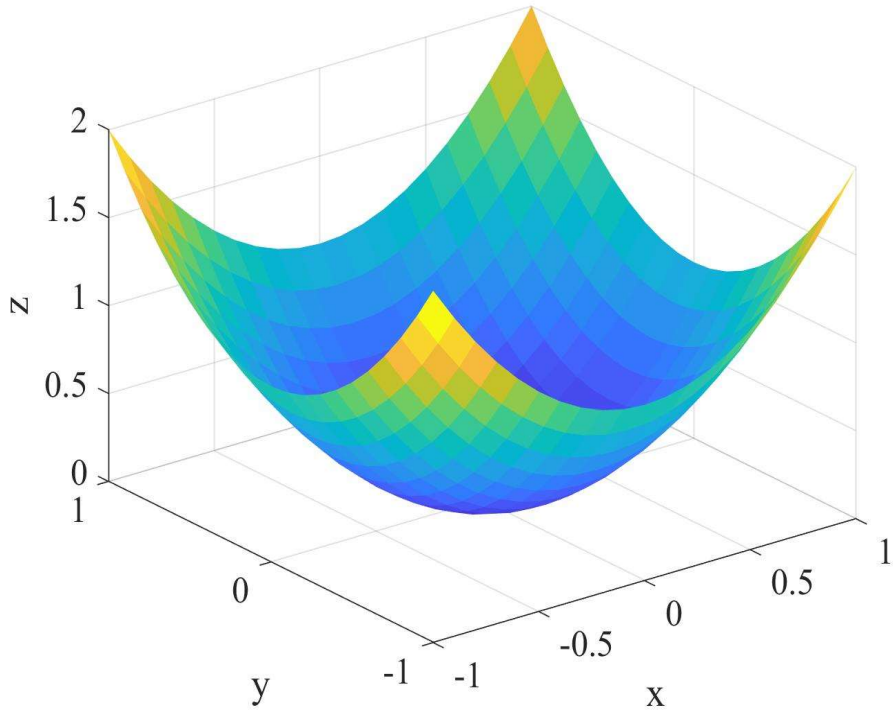


Surface properties

>> s = surf(X,Y,Z)

Turn off the display of the edges

>> s.EdgeColor = 'none';



Surface properties

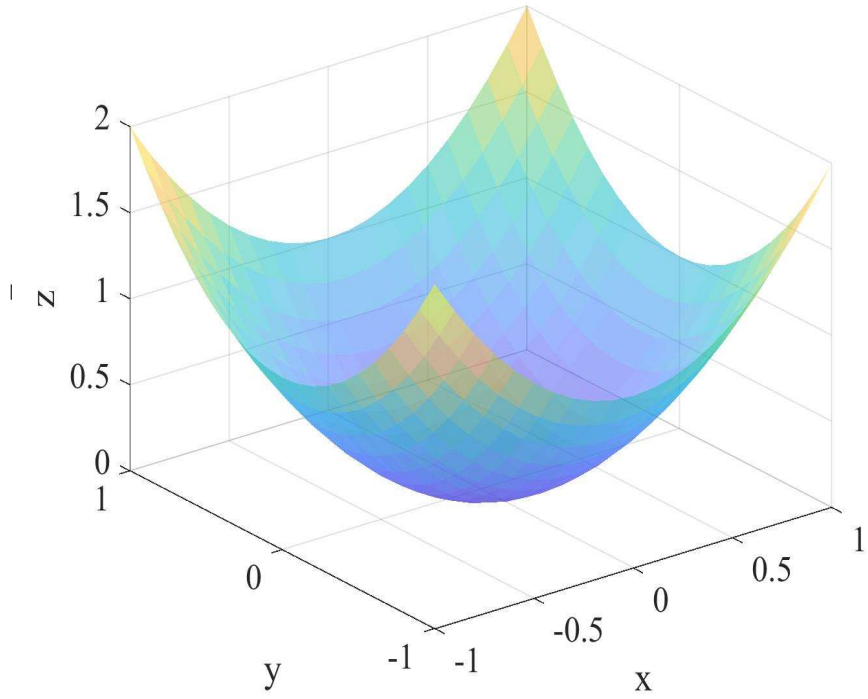
>> s = surf(X,Y,Z)

Turn off the display of the edges

>> s.EdgeColor = 'none';

Create a semitransparent surface

>> s.FaceAlpha = 0.5



Surface properties

>> s = surf(X,Y,Z)

Turn off the display of the edges

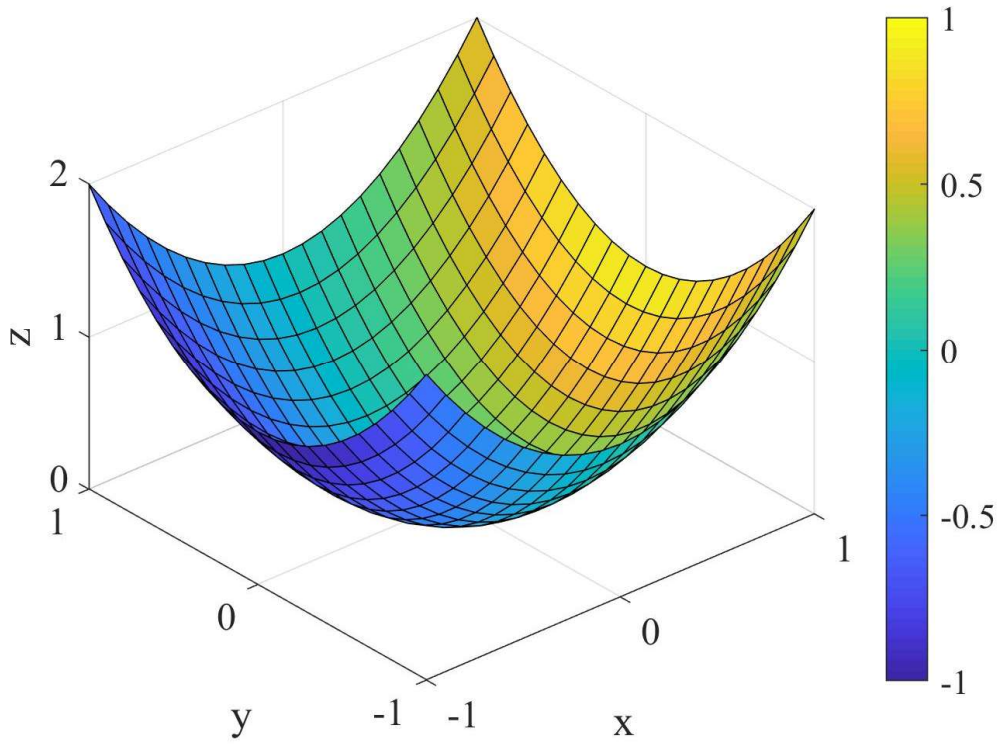
>> s.EdgeColor = 'none';

Create a semitransparent surface

>> s.FaceAlpha = 0.5

Customize the colours of the surface

```
>> C = X.*cos(Y);
>> s = surf(X,Y,Z,C)
>> colorbar
```



Exercise 2

1. Plot the function

$$f(x, y) = x \sin(xy)$$

in the region $0 \leq x \leq 5, \pi \leq y \leq 2\pi$ using the commands **meshgrid** and **surf**

2. Set the surface transparency as 0.6 and delete the edges

3. Rename the figure axis as: 'X', 'Y',' f(X,Y)'.

Exercise 1 (part 2)

2. Approximate the marginal distributions of X and Y using the histograms and draw over them the theoretical marginal distributions.

3. Open a figure with two subplots horizontally aligned and draw in the first one the theoretical probability density function (pdf) of the bivariate normal distribution (obtain the values using **mvnpdf**), from which the data points X and Y were sampled.

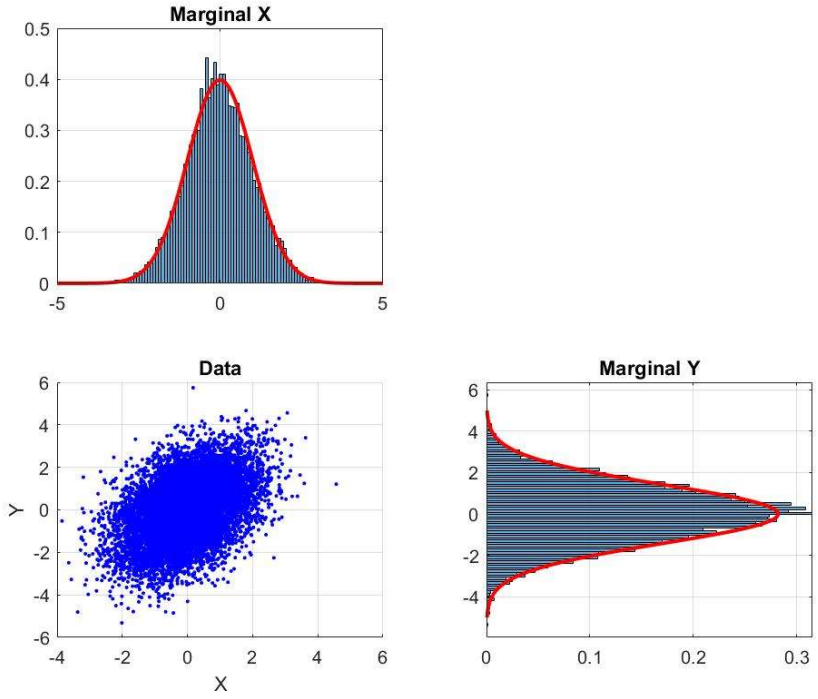
4. Draw in the second subplot the approximation of the theoretical probability density function, using the function **histogram2.**

5. Draw the theoretical cumulative distribution function (cdf) of the bivariate normal distribution (obtain the values using the command **mvncdf**.)

6. **Bonus:** approximate the function drawn in the point 3) using the function **ksdensity.**

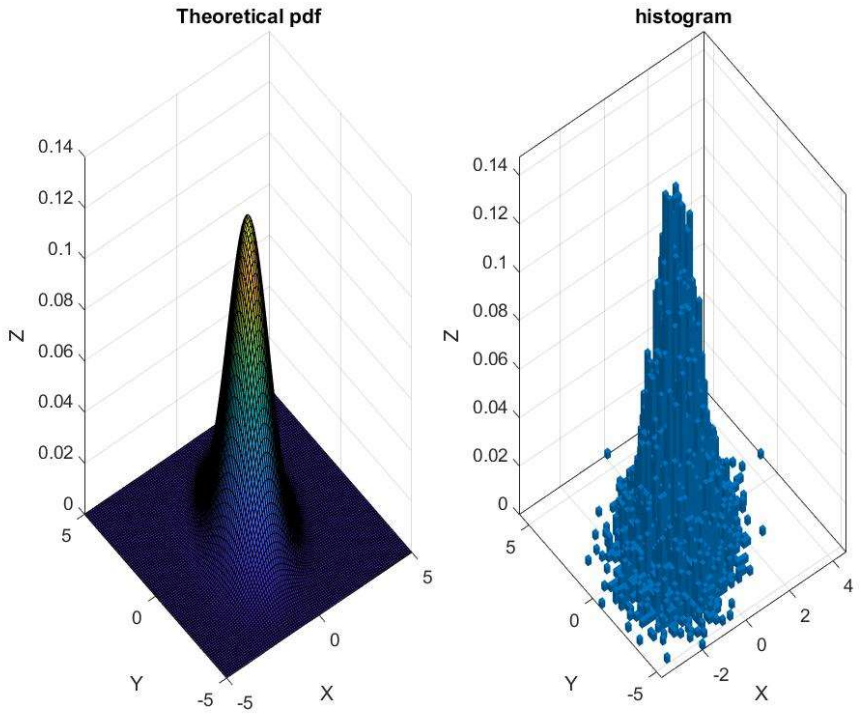
Exercise 1 (part 2): results

2. Result using x_grid = linspace(-5,5,100); y_grid = linspace(-5,5,100); n_bins = 100:



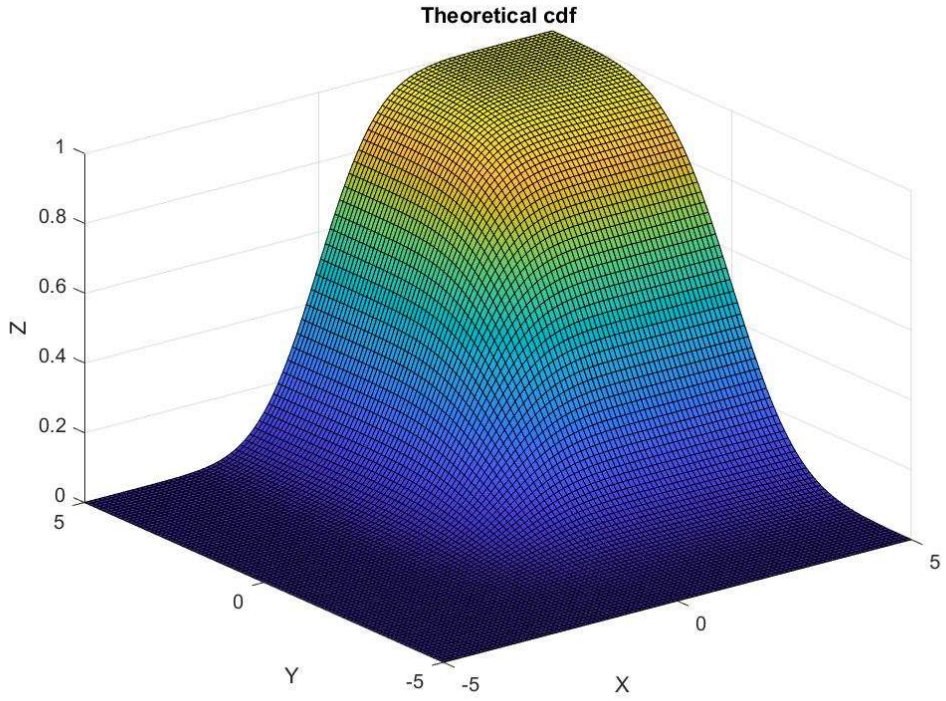
Exercise 1 (part 2): results

3. and 4. Result using n_bins=50:



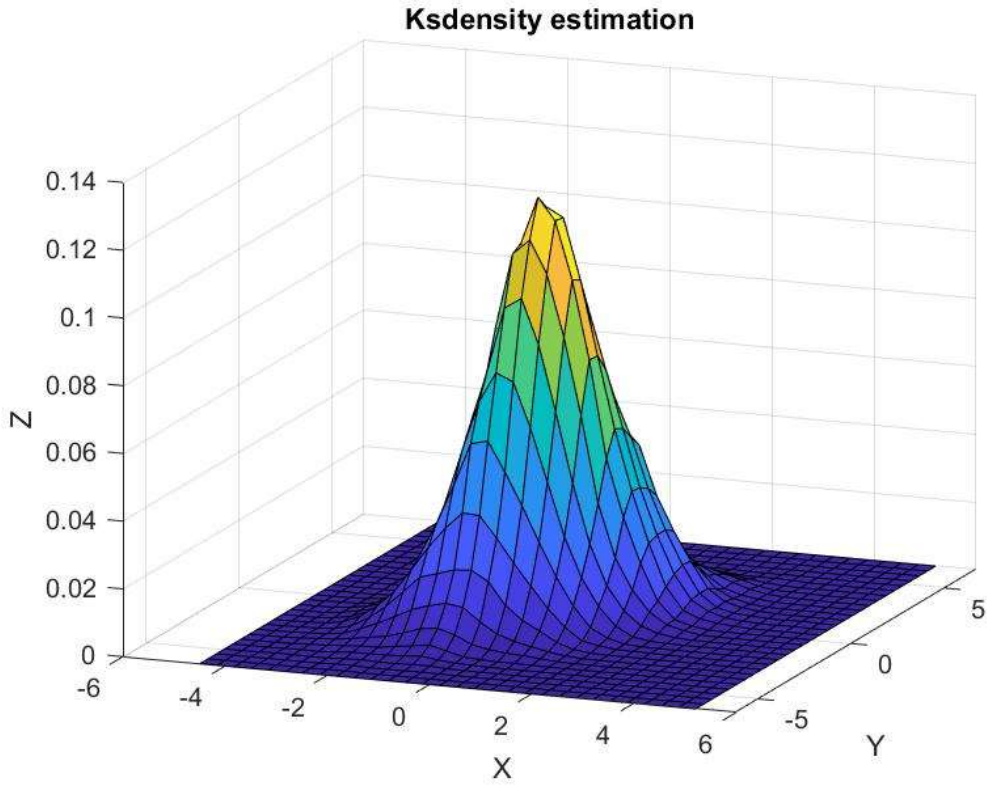
Exercise 1 (part 2): results

5. Result:



Exercise 1 (part 2): results

6. Result:



Reference Documentation:

- https://it.mathworks.com/

- http://sisdin.unipv.it/labsisdin/teaching/courses/imadlt/esercitazioni