Course of Process Control

## Examination

Given a multi-input multi-output (MIMO) process consisting of 3 subsystems with control inputs  $u_1$ ,  $u_2$ , and controlled variables  $y_1$ ,  $y_2$ , one has that  $u_1$  affects  $y_1$  as described by the following equations:

$$\begin{cases} x_1 = x_2 \\ \dot{x}_2 = -10^3 \sin(x_2) - 5\sin(x_2) - \frac{10^4}{2}x_1 - 5 \cdot 10^3 \sin(x_1) - 5x_2 + 10^3 u_1 \\ y_1 = \sin(x_1) \end{cases}$$

where  $x_1$  and  $x_2$  are the subsystem (Subsystem 1) state variables. Variable  $u_1$  also affects  $y_2$  according to the following relationship:

$$Y_2(s) = G_{22}(s)U_2(s) + G_{21}U_1(s)$$

where  $U_1(s)$ ,  $U_2(s)$  and  $Y_2(s)$  are the Laplace Transforms of signals  $u_1(t)$ ,  $u_2(t)$  and  $y_2(t)$ , respectively, while

$$G_{21} = \frac{10^3(s+10^6)}{s+1}, \quad G_{22} = \frac{10^{-3}(s+10^6)}{s+10^2}$$

are the transfer functions of Subsystem 2 e 3. Variable  $u_2$  does not affect  $y_1$ . Let  $y_{1_{rif}}$  and  $y_{2_{rif}}$  be the reference signals.

Design a digital MIMO linear time-invariant (LTI) control system capable of decoupling the MIMO system into a couple of SISO control loops. To this end:

- 1. First, with reference only to Subsystem 1:
  - (a) Determine the constant input  $\bar{u}_1$  such that, at the equilibrium,  $\bar{x}_1 = 0$ .
  - (b) Linearize the subsystem at the equilibrium state previously determined.
  - (c) Determine the input/output transfer function, to be denoted with  $G_{11}(s)$  (transfer function of Subsystem 1).
- 2. Determine the transfer matrix G(s) for the entire process under concern.
- 3. Design a decoupler for the MIMO system.
- 4. For the first control loop (that involving  $u_1$  and  $y_1$ ), design a PI controller capable of attenuating process disturbances acting on the controlled variable, assuming that they have significant harmonics for  $\omega \leq 10 \ rad/s$ , as well as measurement disturbances with significant harmonics for  $\omega \geq 10^3 \ rad/s$ .
- 5. For the first control loop, also design and anti-wind-up scheme.
- 6. For the second control loop (that involving  $u_2$  and  $y_2$ ), design a controller guaranteeing a zero steady state error when the reference signal undergoes a step variation, and providing a closed loop bandwidth of at least 10 rad/s.
- 7. Modify the second control loop so that the final closed loop bandwidth is increased of at least a decade.
- 8. Provide a block scheme of the overall control scheme.
- 9. Determine the sampling pulse  $\omega_s$  such that at the corresponding Nyquist pulse the frequency responses of the two SISO loops present at least 20dB of attenuation.
- 10. Determine, for the two loops, the phase margin reduction due to the presence, in each loop, of the zero-order-hold.
- 11. Discretize the two SISO controllers using Tustin method, discussing stability issues.