Examination

Exercise 1

Given a multi-input multi-output (MIMO) process consisting of 3 subsystems with control inputs u_1, u_2, and controlled variables y_1, y_2, one has that u_1 affects y_1 as described by the following equations:

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -10^3 sin(x_2) - 5 sin(x_2) - \frac{10^4}{2} x_1 - 5 \cdot 10^3 sin(x_1) - 5x_2 + 10^3 u_1 \\ y_1 &= sin(x_1) \end{cases}$$

where x_1 and x_2 are the subsystem (Subsystem 1) state variables. Variable u_1 also affects y_2 according to the following relationship:

$$Y_2(s) = G_{22}(s)U_2(s) + G_{21}U_1(s)$$

where $U_1(s)$, $U_2(s)$ and $Y_2(s)$ are the Laplace Transforms of signals $u_1(t)$, $u_2(t)$ and $y_2(t)$, respectively, while

$$G_{21} = \frac{10^3(s + 10^6)}{s + 1}, \quad G_{22} = \frac{10^{-3}(s + 10^6)}{s + 10^2}$$

are the transfer functions of Subsystem 2 e 3. Variable u_2 does not affect y_1. Let $y_{1_{rif}}$ and $y_{2_{rif}}$ be the reference signals.

Design a digital MIMO linear time-invariant (LTI) control system capable of decoupling the MIMO system into a couple of SISO control loops. To this end:

1. First, with reference only to Subsystem 1:
   (a) Determine the constant input $\bar{u}_1$ such that, at the equilibrium, $\bar{x}_1 = 0$.
   (b) Linearize the subsystem at the equilibrium state previously determined.
   (c) Determine the input/output transfer function, to be denoted with $G_{11}(s)$ (transfer function of Subsystem 1).
2. Determine the transfer matrix $G(s)$ for the entire process under concern.
3. Design a decoupler for the MIMO system.
4. For the first control loop (that involving u_1 and y_1), design a PI controller capable of attenuating process disturbances acting on the controlled variable, assuming that they have significant harmonics for $\omega \leq 10\ rad/s$, as well as measurement disturbances with significant harmonics for $\omega \geq 10^3\ rad/s$.
5. For the first control loop, also design and anti-wind-up scheme.
6. For the second control loop (that involving u_2 and y_2), design a controller guaranteeing a zero steady state error when the reference signal undergoes a step variation.
7. Determine the sampling pulse ω_s.
8. Determine, for the two loops, the phase margin reduction due to the presence, in each loop, of the zero-order-hold.
9. Discretize the two SISO controllers using the Euler Forward method.

Exercise 2

Consider an initial reference frame $O_0 - x_0y_0z_0$. Determine a second reference frame by rotating, counterclockwise, the previous reference frame of 30 degrees around the axis y_0 and, again, turning clockwise the frame obtained of 60 degrees around its axis x. Finally, rotate the reference frame obtained of 90 degrees in the clockwise sense around its axis z. Let $O_3 - x_3y_3z_3$ be the frame obtained at the end of the three rotations.

Compute the rotation matrix that describes the coordinate transformation of a vector expressed in the reference frame $O_3 - x_3y_3z_3$ in the coordinates of the same vector expressed in the frame $O_0 - x_0y_0z_0$.

Exercise 3

Draw a planar manipulator with three degrees of mobility: all the joints are of rotoidal type.

1. Number the joints, determine the reference frames associated with the degrees of mobility, as well as the parameters, according to the Denavit-Hartenberg Convention.
2. Define the joint variables and indicate them on the picture.
3. Determine the homogeneous transformation matrix which describes the direct kinematics of the manipulator.

Exercise 4

Describe the differences between the so-called *direct approach* and *indirect approach* to interaction control. Then, draw an interaction control scheme for robot manipulators realizing an *impedance control*.